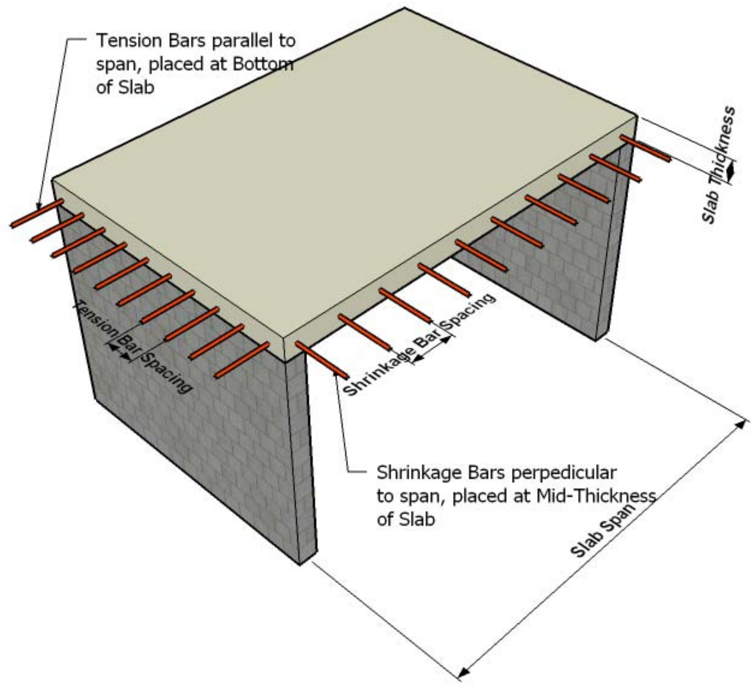
Lecture 6 – One-Way Slabs

A one-way slab is supported by parallel walls or beams, and the main tension reinforcing bars run parallel to the span. It looks like the following:



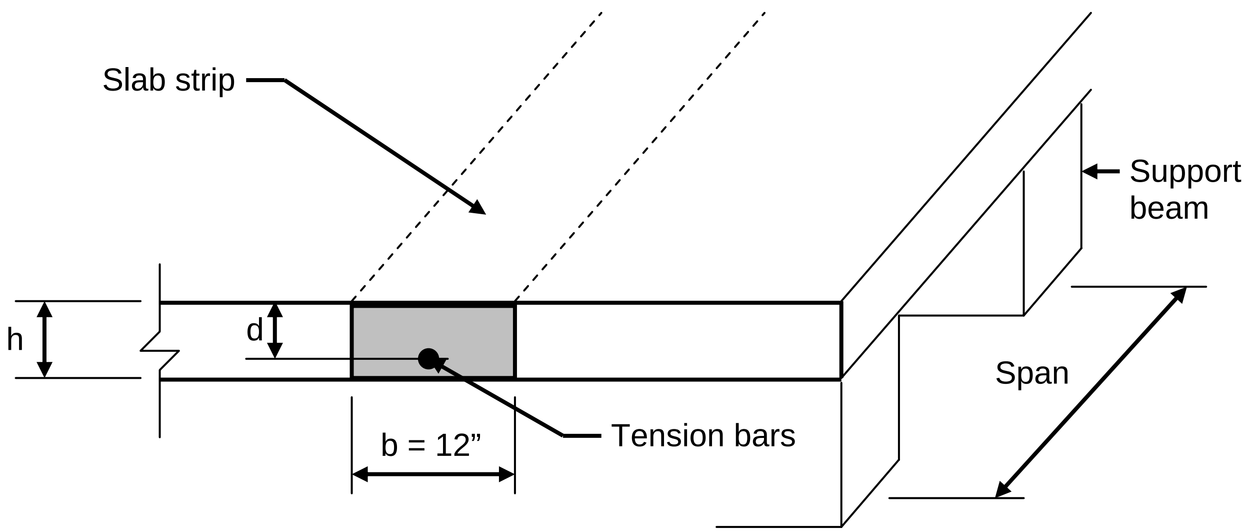
The slab is designed as a series of 1'-0" wide beam "strips". The analysis is similar to rectangular beams, except the width b = 12" and the height is usually on the order of 4" $\rightarrow$ 10". The main tension bars are usually #4, #5 or #6 bars. There are no stirrups in slabs, however, additional bars are placed perpendicular to the main tension bars to prevent cracking during the curing process. These bars are referred to as "shrinkage" or "temperature" bars and are also usually #4 or #5 bars.

Maximum spacing between main **tension bars** = smaller of { Spacing reqd. for moment or 3 x slab thickness or 12" }

Maximum spacing between **shrinkage bars** = smaller of { Spacing reqd. by analysis or 5 x slab thickness or 18" }

Design of Main Tension Bars:

As previously mentioned, slabs are designed as a series of 1'-0" wide rectangular beam "strips" as shown below:



Assuming the slab strip is a rectangular beam, then:

$$M_u = 0.9A_sf_yd\left(1 - \left[0.59\left(\frac{\rho_{act}f_y}{f'_c}\right)\right]\right)$$

where: M_u = Usable moment capacity of slab strip
A_s = Area of tension bars per 1'-0" width of slab
f_y = yield stress of rebar
f'_c = specified compressive strength of concrete
$\rho_{act} = \dfrac{A_s}{bd}$

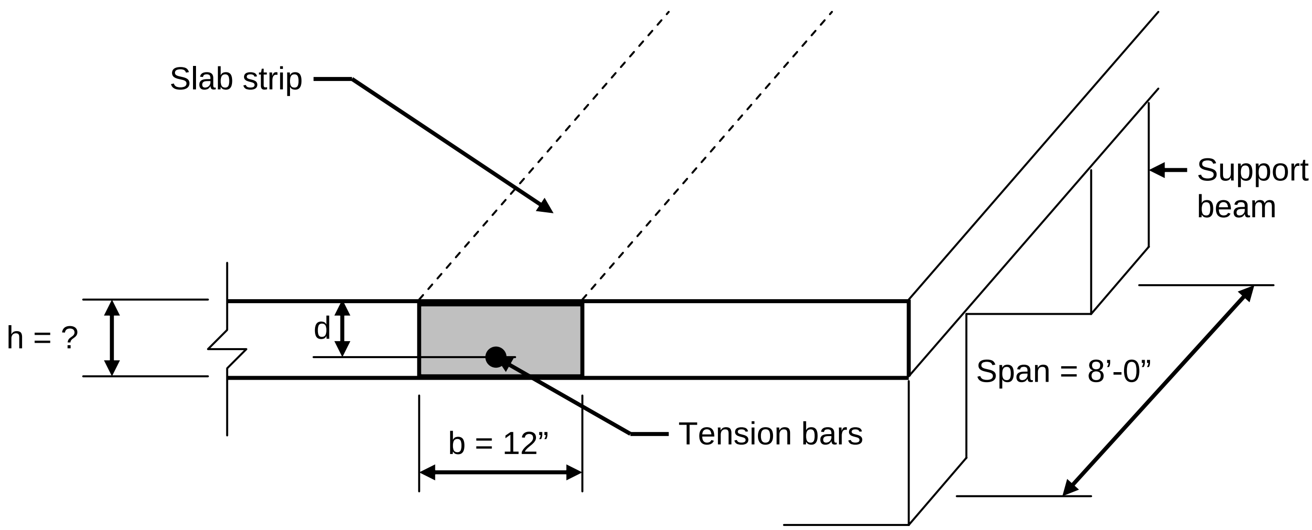
Alternatively, the "Design Aid" Tables 1 and 2 from Lecture 4 may be used for analysis OR design.

Example 1

GIVEN: A one-way slab has a simple span = 8'-0" and the following materials and loads:

- Concrete f'_c = 4000 PSI
- #4 Grade 60 main tension bars and shrinkage bars
- Concrete cover = ¾"
- Superimposed floor SERVICE dead load = 38 PSF (not incl. slab wt.)
- Superimposed floor SERVIVE live load = 125 PSF

REQUIRED: Design the slab, including thickness, main tension bars & shrinkage bars.



Step 1 – Determine slab thickness "h" based on Table below:

Minimum Suggested Thickness "h" of Concrete Beams & One-Way Slabs				
Member:	End Conditions			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Solid one-way slab	L/20	L/24	L/28	L/10
Beam	L/16	L/18.5	L/21	L/8
Span length L = inches				

$$h = \frac{L}{20}$$

$$= \frac{8'\text{–}0''(12''/\ ft)}{20}$$

= 4.8" → **USE 5" Thick Slab**

Step 2 – Determine maximum factored moment, M_{max}, on slab:

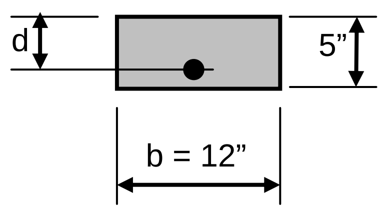
Factored uniform load w_u = 1.2D + 1.6L
= 1.2(superimposed dead load + slab wt.) + 1.6(live load)
= 1.2(1'(38 PSF) + (5/12)(150 PCF)) + 1.6(1'(125 PSF))
= 120.6 PLF + 200 PLF
= 320.6 PLF
= 0.32 KLF

Maximum factored moment $M_{max} = \dfrac{w_u L^2}{8}$

$$= \frac{(0.32 KLF)(8'\text{–}0")^2}{8}$$

M_{max} = 2.56 KIP-FT
= 30.72 KIP-IN
= 30,720 LB-IN

Step 3 – Determine depth to tension bars "d":



d = h" – conc. cover – ½(Tension bar dia.)
= 5" – ¾" – ½(4/8")
= 4"

NOTE: No stirrups are required in slabs

Step 4 – Using Design Aid Table 2 (see Lecture 24), determine A_S:

Determine the corresponding ρ from $\frac{M_u}{\phi b d^2}$:

$$\frac{M_u}{\phi b d^2} = \frac{30{,}720 LB - IN}{(0.9)(12")(4")^2}$$

= 177.8 PSI

	ρ	$\frac{M_u}{\phi b d^2}$
ρ_{min}	0.0033	192.2
	0.0034	197.9
	0.0035	203.5
	0.0036	209.1

Use $\rho_{act} = \rho_{min} = 0.0033$

$\rho_{act} = \frac{A_s}{bd}$

Solve for A_s:

$A_s = \rho_{act}(b)(d)$

$= 0.0033(12")(4")$

$= 0.16\ in^2$ per 1'-0" width of slab

Step 5 – Determine #4 bar tension bar spacing reqd. for moment:

Bar spacing reqd. = $12"\left(\frac{A_s\_per\_one\_bar}{A_s}\right)$

$= 12"\left(\frac{0.20 in^2\_per\_\#4\_bar}{0.16 in^2}\right)$

= 15" apart

Step 6 – Determine MAXIMUM tension bar spacing requirements per ACI-318:

Maximum spacing between main **tension bars** = smaller of
- Spacing reqd. for moment
- or
- 3 x slab thickness
- or
- 12”

= smaller of
- 15” (see prev. page)
- or
- 3 x 5” = 15”
- or
- **12”** ← **USE**

Use #4 Tension bars @ 12” o.c.

Step 7 – Determine “Shrinkage bar requirements by analysis”:

Shrinkage bar A_s = 0.0020bh

= 0.0020(12”)(5”)

= 0.12 in^2 per 1’-0” width

Bar spacing reqd. = $12" \left(\frac{A_s\_per\_one\_bar}{A_s} \right)$

$= 12" \left(\frac{0.20in^2\_per\_\#4\_bar}{0.12in^2} \right)$

= 20” apart

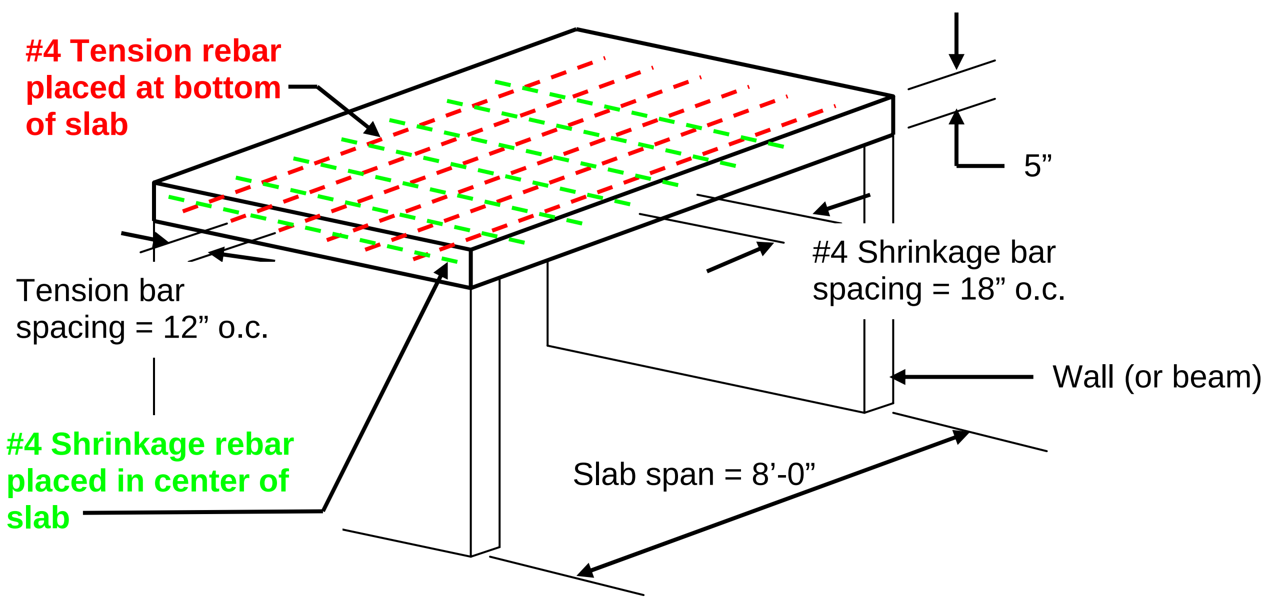
Step 8 – Determine MAXIMUM shrink. bar spacing requirements per ACI-318:

Maximum spacing between **shrinkage bars** = smaller of
- Spacing reqd. by analysis
  or
- 5 x slab thickness
  or
- 18"

= smaller of
- 20" (see prev. pages)
  or
- 5 x 5" = 25"
  or
- **18"** ← **USE**

Use #4 Shrinkage bars @ 18" o.c.

Step 9 – Draw "Summary Sketch":



Continuous (Mult-span) Slabs

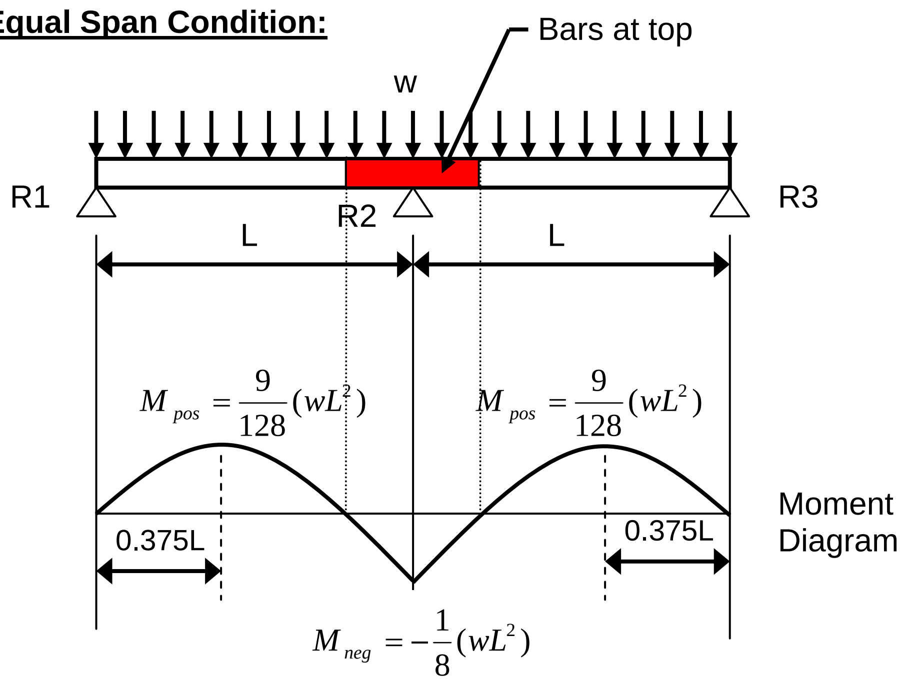
Concrete structural members are typically poured integrally together. Beams and slabs often span multiple supports and are not "simply-supported" as steel and wood framed beams are. As discussed in AECT 360, these concrete beams and slabs are continuous and have both positive moments and negative moments.

The location of tension bars in the members is related to the location of moment:

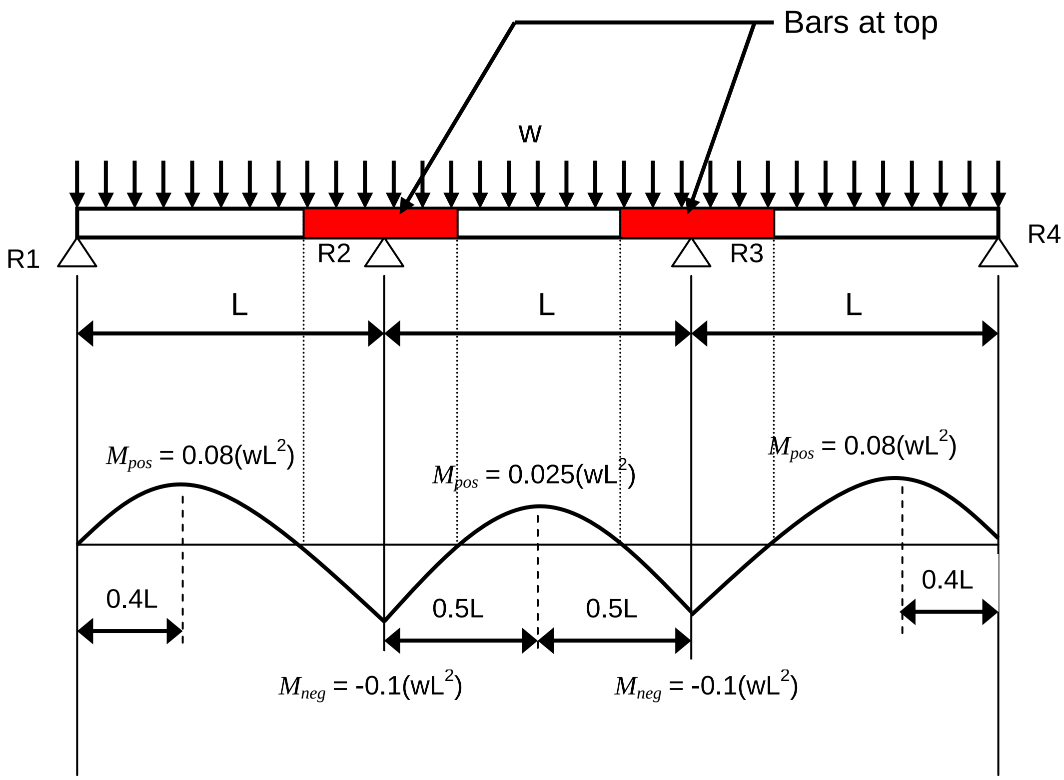
Tension bars are located in the BOTTOM for M_{pos}

Tension bars are located in the TOP for M_{neg}

2- Equal Span Condition:

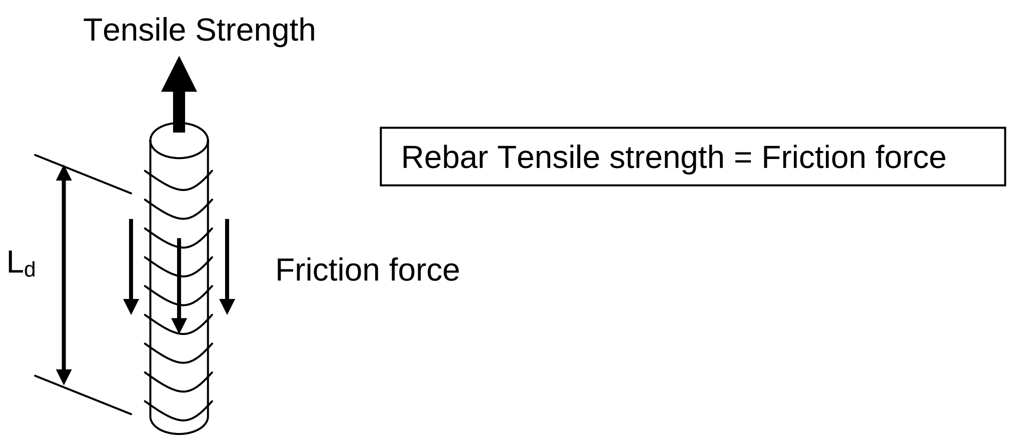


3- Equal Span Condition:

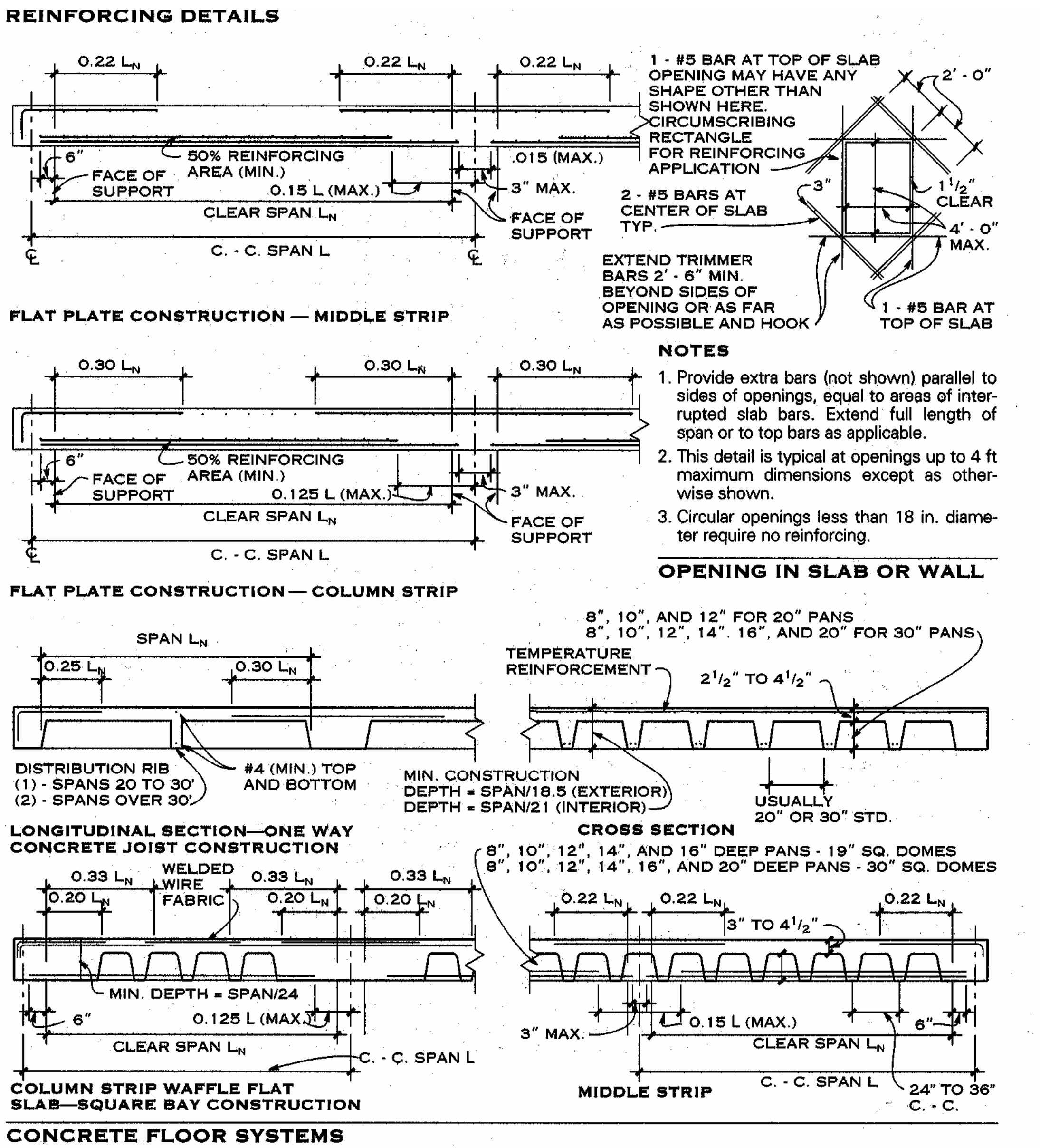


Rebar Placement:

At the transition between the M_{pos} and M_{neg} zones, a minimum overlap of bars is required per ACI 318. These overlaps are required for developing the full bar strength in tension. The friction developed between the concrete and the ribs of the rebar must equal the tensile strength of the bar. The necessary length of the bar embedment to achieve this friction force is called the "Development Length", L_d, and is specified as a multiple of bar diameters. For example, the L_d for a Grade 60 rebar and concrete f'_c = 4000 PSI = 38 x bar diameter.



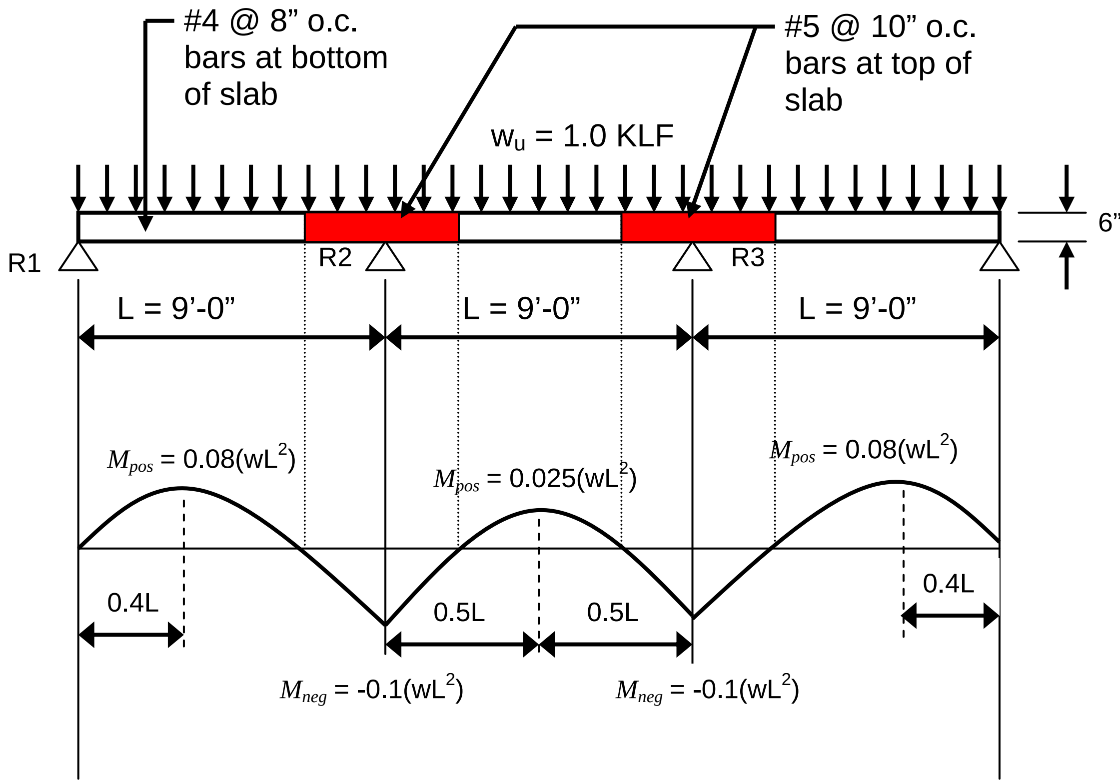
Below are schematic cross-sections of required overlap dimensions for bar placement in continuous slabs (beams are similar):



Example 2

<u>GIVEN</u>: A 1'-0" wide "strip" concrete slab that is 6" thick and a 3-span condition is shown below. All loads shown are already factored and includes slab weight. Use concrete f'_c = 4000 PSI and Grade 60 bars. Use "d" = 5".

<u>REQUIRED</u>: Determine if the slab reinforcing steel is adequate for both the positive moments and negative moments.



<u>Step 1 – Determine maximum factored POSITIVE moment, M_{pos}:</u>

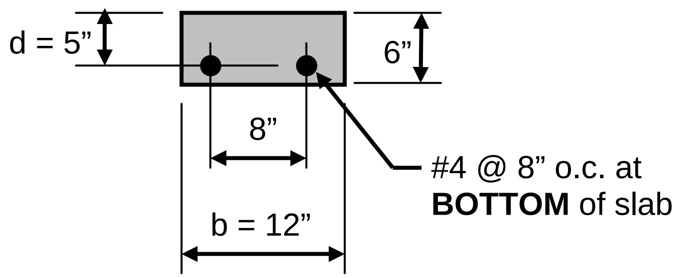
From above, $M_{pos} = 0.08(wL^2)$
$= 0.08(1.0 \text{ KLF})(9'\text{-}0")^2$
$= 6.48$ KIP-FT

<u>Step 2 - Determine maximum factored NEGATIVE moment, M_{neg}:</u>

From above, $M_{pos} = -0.1(wL^2)$
$= -0.1(1.0 \text{ KLF})(9'\text{-}0")^2$
$= -8.1$ KIP-FT

Step 3 – Determine usable moment capacity of slab in POSITIVE moment:

The slab is reinforced with #4 @ 8" o.c.:



We must get the reinforcement A_s in terms of 12" width of slab:

A_s per 1'-0" width = $12"\left(\frac{A_s\_per\_bar}{Spacing}\right)$

$= 12"\left(\frac{0.20in^2\_per\_\#4\_bar}{8"}\right)$

$A_s = 0.30\ in^2$ per 1'-0" width of slab

Determine $\rho_{act} = \frac{A_s}{bd}$

$= \frac{0.30in^2}{(12")(5")}$

$= 0.005$

Determine M_u by formula:

$M_u = 0.9A_sf_yd\left(1 - \left[0.59\left(\frac{\rho_{act} f_y}{f'_c}\right)\right]\right)$

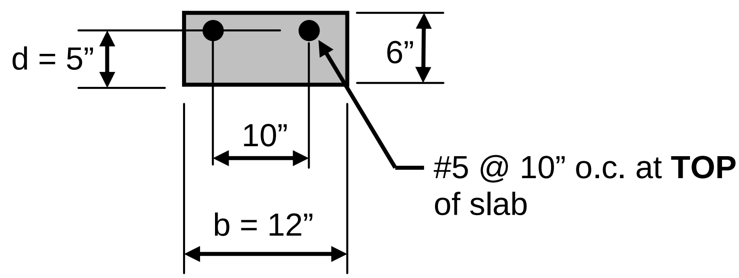
$= 0.9(0.30\ in^2)(60\ KSI)(5")\left(1 - \left[0.59\left(\frac{(0.005)(60)}{4KSI}\right)\right]\right)$

$= 77.4$ KIP-IN

M_u = 6.45 KIP-FT ≈ M_{pos} = 6.48 KIP-FT → ACCEPTABLE

Step 4 – Determine usable moment capacity of slab in NEGATIVE moment:

The slab is reinforced with #5 @ 10" o.c.:



We must get the reinforcement A_s in terms of 12" width of slab:

A_s per 1'-0" width = 12" $\left(\frac{A_s\_per\_bar}{Spacing}\right)$

= 12" $\left(\frac{0.31in^2\_per\_\#5\_bar}{10''}\right)$

A_s = 0.37 in^2 per 1'-0" width of slab

Determine $\rho_{act} = \frac{A_s}{bd}$

$= \frac{0.37in^2}{(12'')(5'')}$

= 0.0061

Determine M_u by formula:

$M_u = 0.9A_sf_yd(1 - \left[0.59\left(\frac{\rho_{act}f_y}{f'_c}\right)\right])$

= 0.9(0.37 in^2)(60 KSI)(5")(1 - $\left[0.59\left(\frac{(0.0061)(60)}{4KSI}\right)\right]$)

= 94.5 KIP-IN

M_u = 7.88 KIP-FT < M_{neg} = 8.1 KIP-FT → NOT ACCEPTABLE