

Lecture 4 – Flexural Members (cont.)

Determining the usable moment capacity, M_u , of a rectangular reinforced concrete beam is accomplished by using the formula below: (see Lect. 3)

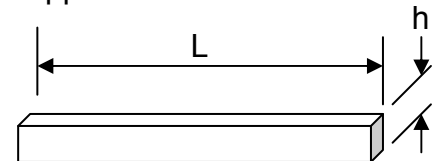
$$M_u = 0.9A_s f_y d \left(1 - 0.59 \left(\frac{\rho_{act} f_y}{f'_c} \right) \right)$$

Designing a beam using the equation above is much more difficult. Assuming the material properties and dimensions are known, the equation above still has 2 unknown variables – A_s and ρ_{act} . Therefore, design of steel reinforcement for a given beam is largely one of trial-and-error.

Beam Design

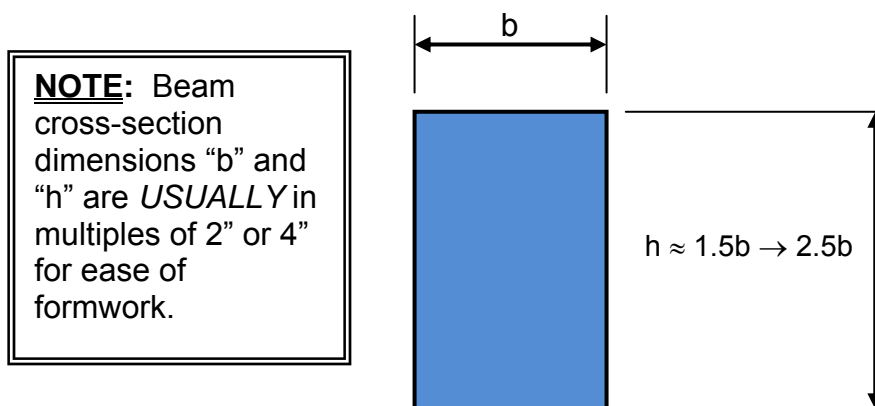
Design of concrete beam members is often one of trial-and-error. It's impossible to directly solve for all the variables in a reinforced concrete beam. Usually, material properties are known as well as maximum applied factored moment, M_{max} .

The following Table is useful to get a “trial” beam size:



Minimum Suggested Thickness “h” of Concrete Beams & One-Way Slabs				
Member:	End Conditions			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Solid one-way slab	L/20	L/24	L/28	L/10
Beam	L/16	L/18.5	L/21	L/8
Span length L = inches				

Beams are usually rectangular having the width typically narrower than the height. The diagram below shows typical beam aspect ratios:



Beam Design Aid

It is still difficult to directly design a reinforced concrete beam even if dimensions and material properties are known. The use of design aids are commonly used to streamline the design process instead of laboriously using a trial-and-error approach.

The design aid shown below is used for design or analysis. Values of $\frac{M_u}{\phi b d^2}$ are in units of PSI. It can be used to directly solve for ρ_{act} knowing factored actual moment M_u , f'_c , f_y , b and d .

	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$
ρ_{min}	0.0033	190.3	0.0066	365.2	0.0099	524.6	0.0131	664.5
	0.0034	195.8	0.0067	370.2	0.0100	529.2	0.0132	668.6
	0.0035	201.3	0.0068	375.3	0.0101	533.8	0.0133	672.8
	0.0036	206.8	0.0069	380.3	0.0102	538.3	0.0134	676.9
	0.0037	212.3	0.0070	385.3	0.0103	542.9	0.0135	681.0
	0.0038	217.8	0.0071	390.3	0.0104	547.4	0.0136	685.0
	0.0039	223.2	0.0072	395.3	0.0105	551.9	0.0137	689.1
	0.0040	228.7	0.0073	400.3	0.0106	556.4	0.0138	693.2
	0.0041	234.1	0.0074	405.2	0.0107	560.9	0.0139	697.2
	0.0042	239.5	0.0075	410.2	0.0108	565.4	0.0140	701.2
	0.0043	244.9	0.0076	415.1	0.0109	569.9	0.0141	705.2
	0.0044	250.3	0.0077	420.0	0.0110	574.3	0.0142	709.2
	0.0045	255.7	0.0078	424.9	0.0111	578.8	0.0143	713.2
	0.0046	261.0	0.0079	429.8	0.0112	583.2	0.0144	717.2
	0.0047	266.4	0.0080	434.7	0.0113	587.6	0.0145	721.1
	0.0048	271.7	0.0081	439.5	0.0114	592.0	0.0146	725.1
	0.0049	277.0	0.0082	444.4	0.0115	596.4	0.0147	729.0
	0.0050	282.3	0.0083	449.2	0.0116	600.7	0.0148	732.9
	0.0051	287.6	0.0084	454.0	0.0117	605.1	0.0149	736.8
	0.0052	292.9	0.0085	458.8	0.0118	609.4	0.0150	740.7
	0.0053	298.1	0.0086	463.6	0.0119	613.7	0.0151	744.6
	0.0054	303.4	0.0087	468.4	0.0120	618.0	0.0152	748.4
	0.0055	308.6	0.0088	473.2	0.0121	622.3	0.0153	752.3
	0.0056	313.8	0.0089	477.9	0.0122	626.6	0.0154	756.1
	0.0057	319.0	0.0090	482.6	0.0123	630.9	0.0155	759.9
	0.0058	324.2	0.0091	487.4	0.0124	635.1	0.0156	763.7
	0.0059	329.4	0.0092	492.1	0.0125	639.4	0.0157	767.5
	0.0060	334.5	0.0093	496.8	0.0126	643.6	0.0158	771.2
	0.0061	339.7	0.0094	501.4	0.0127	647.8	0.0159	775.0
	0.0062	344.8	0.0095	506.1	0.0128	652.0	0.0160	778.7
	0.0063	349.9	0.0096	510.7	0.0129	656.2	ρ_{max}	0.0161
0.0064	355.0	0.0097	515.4	0.0130	660.9			
0.0065	360.1	0.0098	520.0					

Table 1 - Concrete $f'_c = 3000$ PSI, Grade 60 Bars

	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$	ρ	$\frac{M_u}{\phi b d^2}$
ρ_{min}	0.0033	192.2	0.0079	440.9	0.0125	667.0	0.0171	870.7
	0.0034	197.9	0.0080	446.0	0.0126	671.7	0.0172	874.9
	0.0035	203.5	0.0081	451.2	0.0127	676.3	0.0173	879.1
	0.0036	209.1	0.0082	456.3	0.0128	681.0	0.0174	883.2
	0.0037	214.7	0.0083	461.4	0.0129	685.6	0.0175	887.4
	0.0038	220.3	0.0084	466.5	0.0130	690.3	0.0176	891.5
	0.0039	225.9	0.0085	471.6	0.0131	694.9	0.0177	895.6
	0.0040	231.5	0.0086	476.7	0.0132	699.5	0.0178	899.7
	0.0041	237.1	0.0087	481.8	0.0133	704.1	0.0179	903.9
	0.0042	242.6	0.0088	486.9	0.0134	708.6	0.0180	907.9
	0.0043	248.2	0.0089	491.9	0.0135	713.2	0.0181	912.0
	0.0044	253.7	0.0090	497.0	0.0136	717.8	0.0182	916.1
	0.0045	259.2	0.0091	502.0	0.0137	722.3	0.0183	920.2
	0.0046	264.8	0.0092	507.1	0.0138	726.9	0.0184	924.2
	0.0047	270.3	0.0093	512.1	0.0139	731.4	0.0185	928.3
	0.0048	275.8	0.0094	517.1	0.0140	735.9	0.0186	932.3
	0.0049	281.2	0.0095	522.1	0.0141	740.4	0.0187	936.3
	0.0050	286.7	0.0096	527.1	0.0142	744.9	0.0188	940.3
	0.0051	292.2	0.0097	532.0	0.0143	749.4	0.0189	944.3
	0.0052	297.6	0.0098	537.0	0.0144	753.9	0.0190	948.3
	0.0053	303.1	0.0099	542.0	0.0145	758.3	0.0191	952.3
	0.0054	308.5	0.0100	546.9	0.0146	762.8	0.0192	956.2
	0.0055	313.9	0.0101	551.8	0.0147	767.2	0.0193	960.2
	0.0056	319.3	0.0102	556.7	0.0148	771.7	0.0194	964.1
	0.0057	324.7	0.0103	561.7	0.0149	776.1	0.0195	968.1
	0.0058	330.1	0.0104	566.6	0.0150	780.5	0.0196	972.0
	0.0059	335.5	0.0105	571.5	0.0151	784.9	0.0197	975.9
	0.0060	340.9	0.0106	576.3	0.0152	789.3	0.0198	979.8
	0.0061	346.2	0.0107	581.2	0.0153	793.7	0.0199	983.7
	0.0062	351.6	0.0108	586.1	0.0154	798.1	0.0200	987.6
	0.0063	356.9	0.0109	590.9	0.0155	802.4	0.0201	991.5
	0.0064	362.2	0.0110	595.7	0.0156	806.8	0.0202	995.3
	0.0065	367.6	0.0111	600.6	0.0157	811.1	0.0203	999.2
	0.0066	372.9	0.0112	605.4	0.0158	815.4	0.0204	1003.0
	0.0067	378.2	0.0113	610.2	0.0159	819.7	0.0205	1006.8
	0.0068	383.4	0.0114	615.0	0.0160	824.1	0.0206	1010.7
	0.0069	388.7	0.0115	619.8	0.0161	828.3	0.0207	1014.5
	0.0070	394.0	0.0116	624.5	0.0162	832.6	0.0208	1018.3
	0.0071	399.2	0.0117	629.3	0.0163	836.9	0.0209	1022.0
	0.0072	404.5	0.0118	634.1	0.0164	841.2	0.0210	1025.8
	0.0073	409.7	0.0119	638.8	0.0165	845.4	0.0211	1029.6
	0.0074	414.9	0.0120	643.5	0.0166	849.7	0.0212	1033.3
	0.0075	420.1	0.0121	648.2	0.0167	853.9	0.0213	1037.1
	0.0076	425.3	0.0122	653.0	0.0168	858.1	ρ_{max} 0.0214	1040.8
	0.0077	430.5	0.0123	657.7	0.0169	862.3		
	0.0078	435.7	0.0124	662.3	0.0170	866.5		

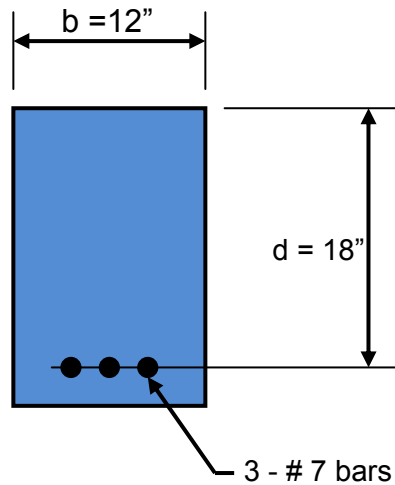
Table 2 – Concrete $f'_c = 4000$ PSI, Grade 60 Bars

Example 1

GIVEN: A rectangular concrete beam with dimensions is shown below (stirrup bars not shown). Use concrete $f'_c = 4000$ PSI and grade 60 bars.

REQUIRED:

- 1) Determine the usable moment capacity M_u of the beam using formula.
- 2) Determine the usable moment capacity M_u of the beam using Table 2.



Step 1 – Determine usable moment capacity M_u of the beam using formula:

$$\begin{aligned}\rho_{act} &= \frac{A_s}{bd} \\ &= \frac{3(0.60 \text{ in}^2 \text{ per bar})}{(12'')(18'')}\end{aligned}$$

$$\rho_{act} = 0.0083$$

$$\begin{aligned}M_u &= 0.9A_s f_y d \left(1 - \left[0.59 \left(\frac{\rho_{act} f_y}{f'_c} \right) \right] \right) \\ &= 0.9(1.80 \text{ in}^2)(60 \text{ KSI})(18'') \left(1 - \left[0.59 \left(\frac{(0.0083)(60 \text{ KSI})}{4 \text{ KSI}} \right) \right] \right) \\ &= 1621 \text{ KIP-IN}\end{aligned}$$

$$\boxed{M_u = 135 \text{ KIP-FT}}$$

Step 2 - Determine the usable moment capacity M_u of the beam using Table 2:

From Table 2 above:

$$\text{At } \rho = 0.0083 \rightarrow \frac{M_u}{\phi b d^2} = 461.4 \text{ PSI}$$

Solving for M_u :

$$\begin{aligned} M_u &= 461.4 \text{ PSI}(\phi b d^2) \\ &= 461.4 \text{ PSI}[(0.9)(12'')(18'')^2] \\ &= 1,614,531 \text{ LB-IN} \\ &= 1615 \text{ KIP-IN} \end{aligned}$$

$$\boxed{M_u = 134.6 \text{ KIP-FT}}$$

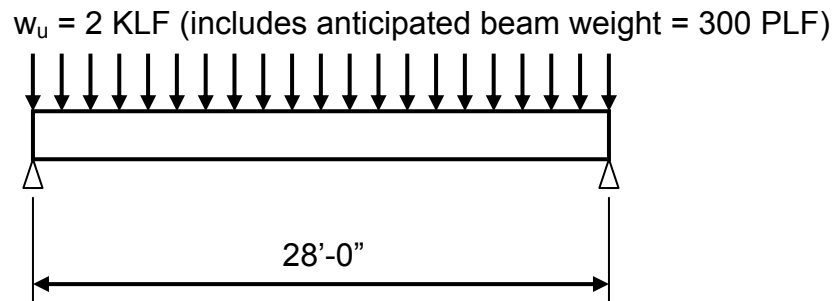
NOTE: This answer is the same as in Step 1.

Example 2

GIVEN: The concrete beam below. Use the following:

- Concrete $f'_c = 4000$ PSI
- Steel grade 60
- Concrete cover = $\frac{3}{4}$ "
- #8 bars are to be used for main tension bars
- #3 stirrups

REQUIRED: Design the rectangular beam such that $h \approx 1.5b$ and $\rho_{act} \approx \frac{1}{2} (\rho_{max})$.



Step 1 – Determine maximum factored moment, M_{max} :

$$\begin{aligned} M_{max} &= \frac{w_u L^2}{8} \\ &= \frac{(2\text{KLF})(28'-0'')^2}{8} \end{aligned}$$

$$\begin{aligned} M_{max} &= 196 \text{ KIP-FT} \\ &= 2352 \text{ KIP-IN} \\ &= 2,352,000 \text{ LB-IN} \end{aligned}$$

Step 2 – Select values from Table 2:

a) Select $\rho_{act} = \frac{1}{2}(\rho_{max})$
 $= \frac{1}{2}(0.0214)$

TRY $\rho_{act} = 0.0107$

b) At $\rho = 0.0107 \rightarrow \frac{M_u}{\phi b d^2} = 581.2 \text{ PSI}$

Step 3 – Solve for “b” and “d” by substituting M_{max} for M_u in above equation:

$$\frac{M_u}{\phi b d^2} = 581.2 \text{ PSI}$$

where: $M_u = M_{max} = 2,352,000 \text{ LB-IN}$

$$\phi = 0.9$$

$$d = 1.5b$$

$$\frac{2352000 \text{ Lb-in}}{(0.9)(b)(1.5b)^2} = 581.2 \text{ PSI}$$

Solve for “b”:

$$\frac{2352000}{(0.9)(2.25b^3)} = 581.2 \text{ PSI}$$

$$b = \sqrt[3]{\frac{2352000}{(0.9)(2.25)(581.2)}}$$

$b = 12.6'' \rightarrow$ **Use b = 12''**

$$d = 1.5b \\ = 1.5(12'')$$

$d \approx 18''$

Step 4 – Select beam dimensions:

From above, use $b = 12''$ and $d \approx 18''$

#3 stirrup bar dia. = $3/8''$  #8 main bar dia. = $1''$ 

$$h = d + \text{conc. cover} + \text{stirrup bar dia.} + \frac{1}{2}(\text{main bar dia.})$$

$$= 18'' + \frac{3}{4}'' + \frac{3}{8}'' + \frac{1}{2}(1'')$$

$$= 19.625'' \rightarrow \text{Use } h = 20''$$

$$\begin{aligned} \text{Revised } d &= 20'' - \frac{3}{4}'' - \frac{3}{8}'' - \frac{1}{2}(1'') \\ &= 18.375'' \end{aligned}$$

Step 5 – Determine required area of main tension bars:

$$\text{From above, } \rho_{\text{act}} = 0.0107 = \frac{A_s}{bd}$$

Solve for A_s :

$$A_s = 0.0107(b)(d)$$

$$= 0.0107(12'')(18.375'')$$

$$\underline{A_s = 2.36 \text{ in}^2}$$

Step 6 – Determine number of #8 main tension bars:

$$\text{No. of bars} = \frac{A_s}{\text{Area of one bar}}$$

$$= \frac{2.36 \text{ in}^2}{0.79 \text{ in}^2 \text{ per \#8 bar}}$$

$$= 2.99 \text{ bars} \rightarrow \text{USE 3 - \#8 bars}$$

Step 7 – Check beam height with “Minimum Thickness of Beams” Table:

From Table:

Member type = Beam

End Condition = Simply-supported

$$h \approx \frac{L}{16}$$

$$h \approx \frac{L}{16}$$

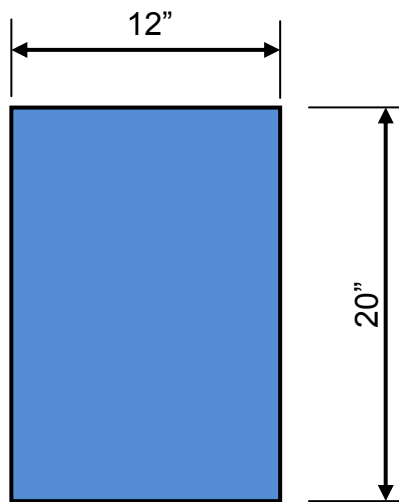
$$\approx \frac{(28'-0'')(12'' / ft)}{16}$$

$h \approx 21''$ which is approximately = 20'' as designed

Step 8 – Check originally assumed beam weight:

It was **assumed** that the beam weight = 300 PLF (See Step 1).

Actual beam dimensions:

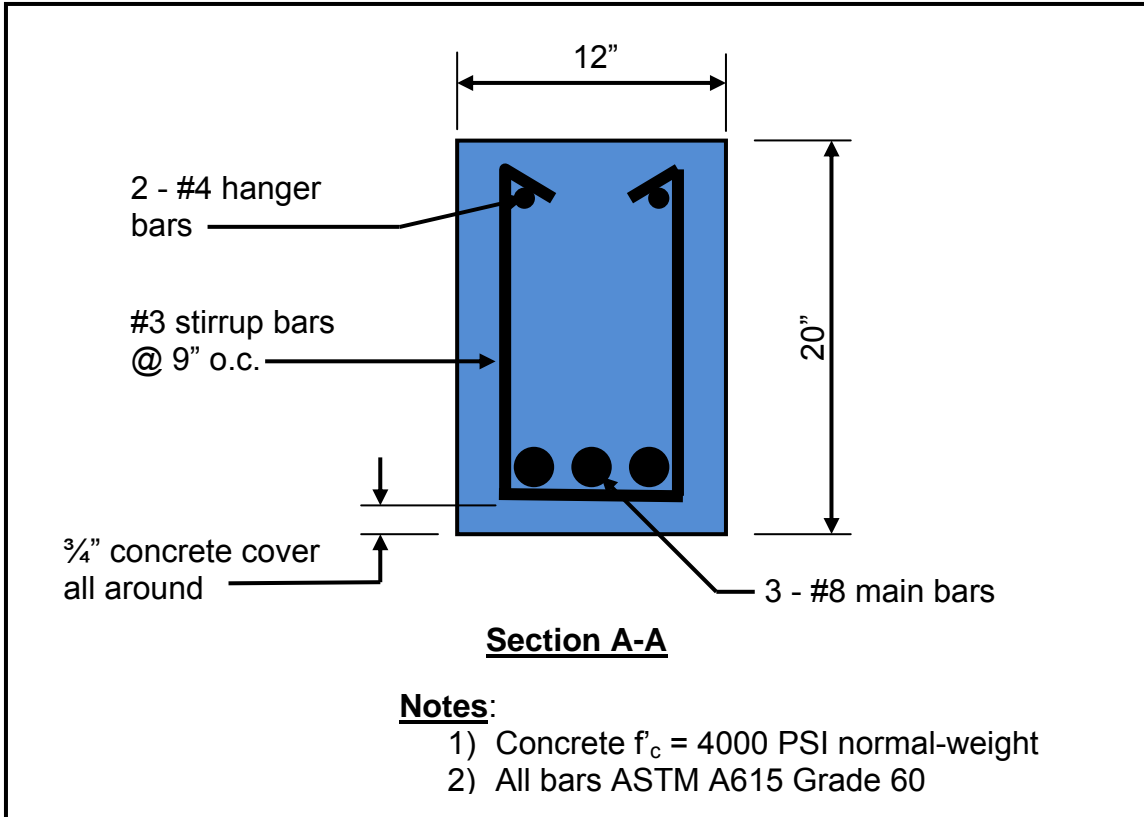


Recall: $UnitWt = \frac{Weight}{Volume}$ → Rearranging:

$$\begin{aligned} BeamWeight &= (UnitWt)(Volume) \\ &= (150 PCF) \left(\frac{12'' \times 20''}{144in^2 / ft^2} \right) (1'-0'') \end{aligned}$$

Beam Weight = 250 PLF < 300 PLF (assumed) OK

Step 9 – Draw “Summary Sketch” labeling all information necessary to build it:



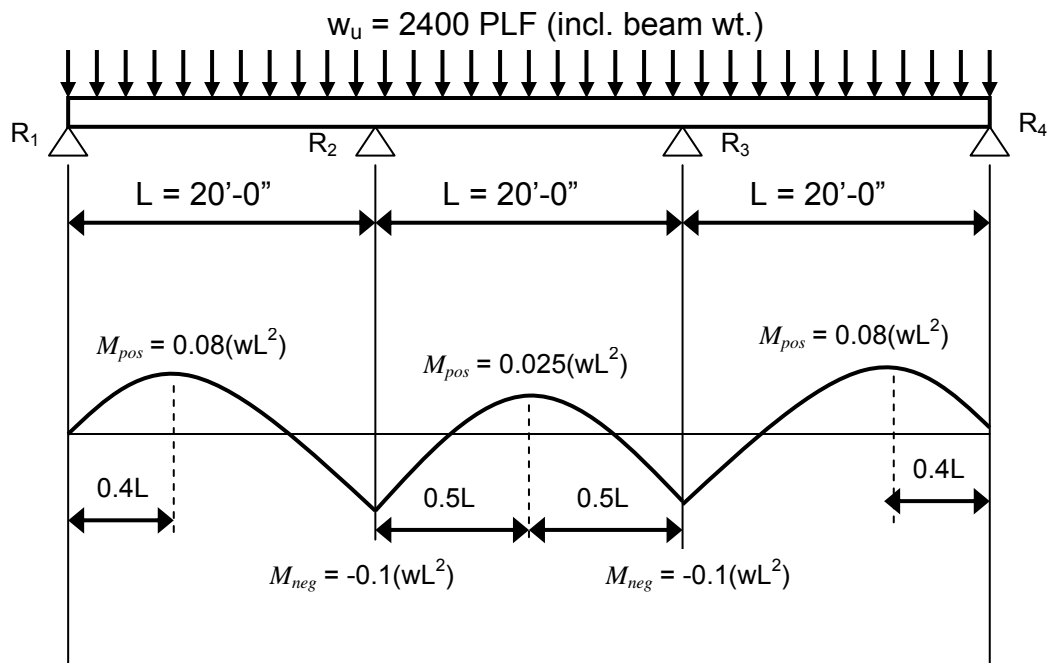
Example 3

GIVEN: A continuous 10" x 16" concrete beam is shown below. Assume the following:

- Concrete $f'_c = 4000$ PSI
- Steel grade 60
- Concrete cover = $\frac{3}{4}$ "
- 2 - #9 bars are to be used for main tension bars
- #3 stirrups

REQUIRED:

- 1) Determine the magnitude of the maximum positive and negative moments (Refer to Structural Theory notes)
- 2) Determine if the beam is adequate based upon flexure only. Check positive & negative moments.



$$R_1 = 0.4wL$$

$$R_2 = 1.1wL$$

$$R_3 = 1.1wL$$

$$R_4 = 0.4wL$$

Step 1 – Determine maximum positive and maximum negative moments:

a) Maximum positive moment, “M_{pos}”:

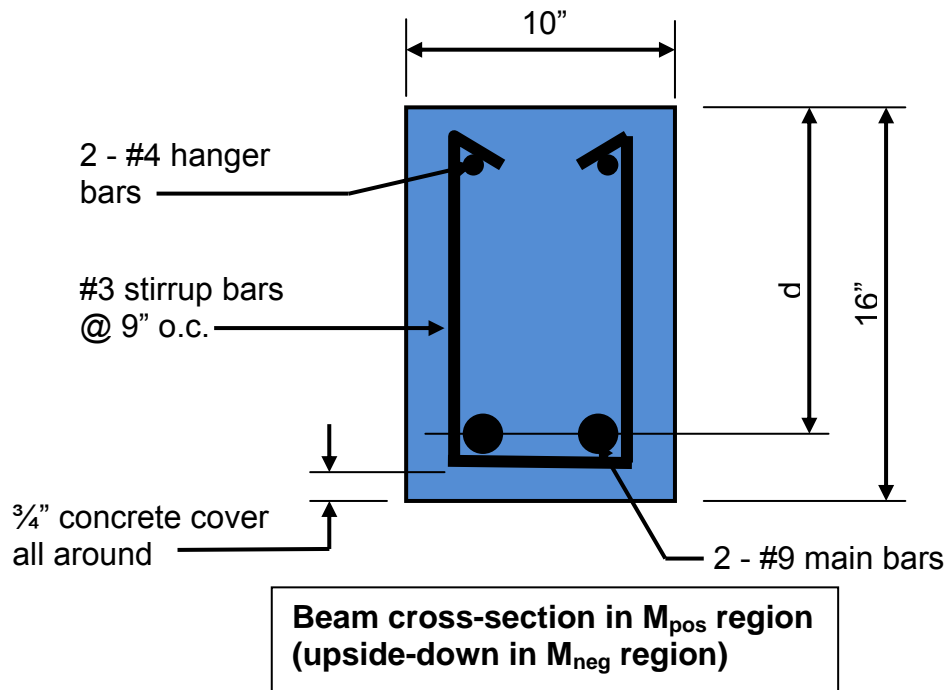
$$\begin{aligned} \text{From above, the largest } M_{\text{pos}} &= 0.08(wL^2) \\ &= 0.08(2.4 \text{ KLF})(20'-0")^2 \\ &= 76.8 \text{ Kip-Ft} \end{aligned}$$

b) Maximum negative moment, “M_{neg}”:

$$\begin{aligned} \text{From above, the largest } M_{\text{neg}} &= -0.1(wL^2) \\ &= -0.1(2.4 \text{ KLF})(20'-0")^2 \\ &= 96.0 \text{ Kip-Ft} \end{aligned}$$

Step 2 – Determine beam’s “d” dimension:

Depth “d” to center of tension steel is shown below:



$$\begin{aligned} \#3 \text{ stirrup bar dia.} &= 3/8" & \#9 \text{ main bar dia.} &= \left(\frac{9}{8}\right) \end{aligned}$$

$$d = h - \text{conc. cover} - \text{stirrup bar dia.} - \frac{1}{2}(\text{main bar dia.})$$

$$= 16" - \frac{3}{4}" - \frac{3}{8}" - \frac{1}{2}\left(\frac{9}{8}\right)$$

d = 14.3125"

Step 3 – Determine usable moment capacity M_u of the beam using formula:

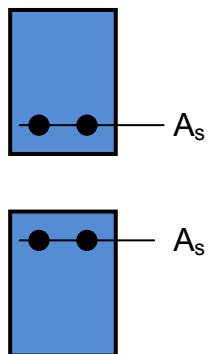
$$\rho_{act} = \frac{A_s}{bd}$$
$$= \frac{2(1.00in^2 \text{ _ per _ bar})}{(10")(14.3125")}$$

$\rho_{act} = 0.01397$ which is $> \rho_{min} = 0.0033$ and $< \rho_{max} = 0.0214$ **OK**

$$M_u = 0.9A_s f_y d \left(1 - \left[0.59 \left(\frac{\rho_{act} f_y}{f'_c} \right) \right] \right)$$
$$= 0.9(2.00 \text{ in}^2)(60 \text{ KSI})(14.3125") \left(1 - \left[0.59 \left(\frac{(0.01397)(60 \text{ KSI})}{4 \text{ KSI}} \right) \right] \right)$$
$$= 1354.6 \text{ KIP-IN}$$

$M_u = 112.9 \text{ KIP-FT}$

Step 4 – Check if beam is adequate based upon flexure only:



a) Maximum positive moment, " M_{pos} ":
From above, the largest $M_{pos} = 76.8 \text{ Kip-Ft} < 112.9 \text{ Kip-Ft}$
ACCEPTABLE

b) Maximum negative moment, " M_{neg} ":
From above, the largest $M_{neg} = -96.0 \text{ Kip-Ft} < 112.9 \text{ Kip-Ft}$
ACCEPTABLE