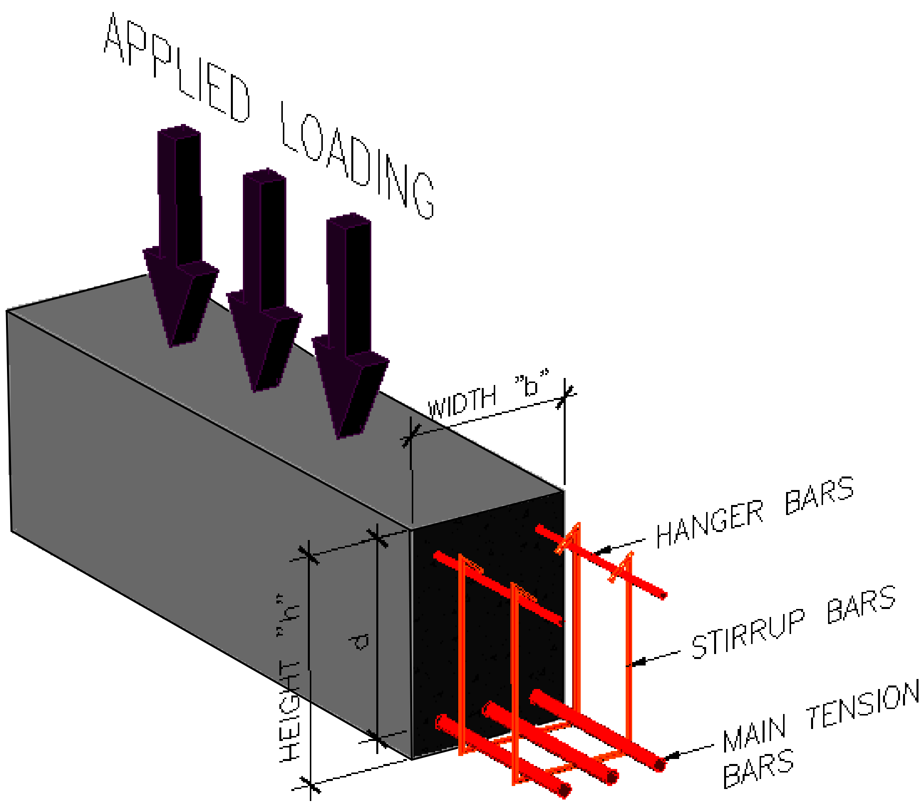
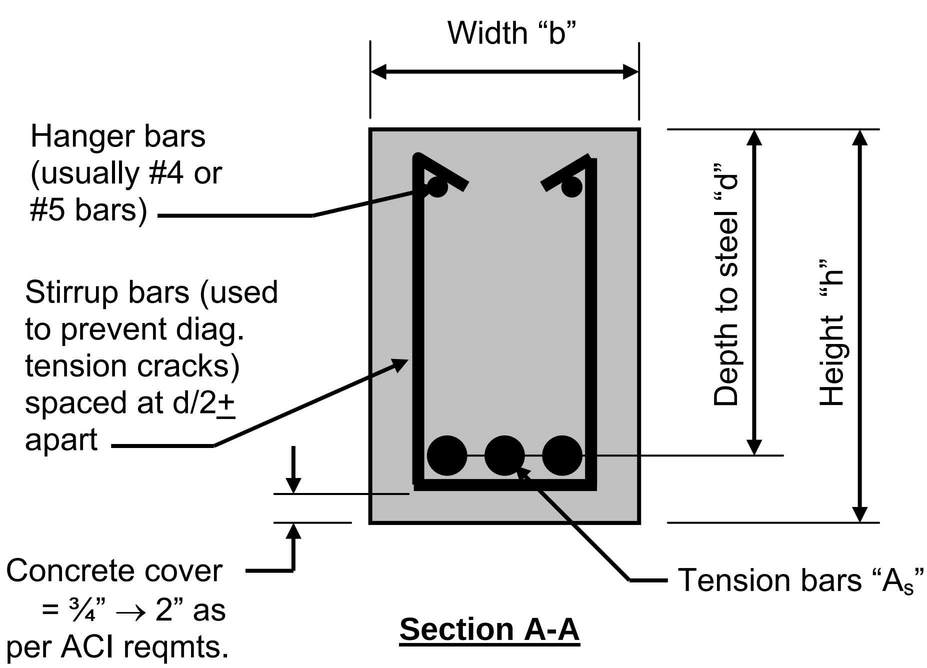
Lecture 3 – Flexural Members

Flexural members are those that experience primarily **bending** stresses, such as beams. A typical rectangular reinforced concrete beam is shown below:

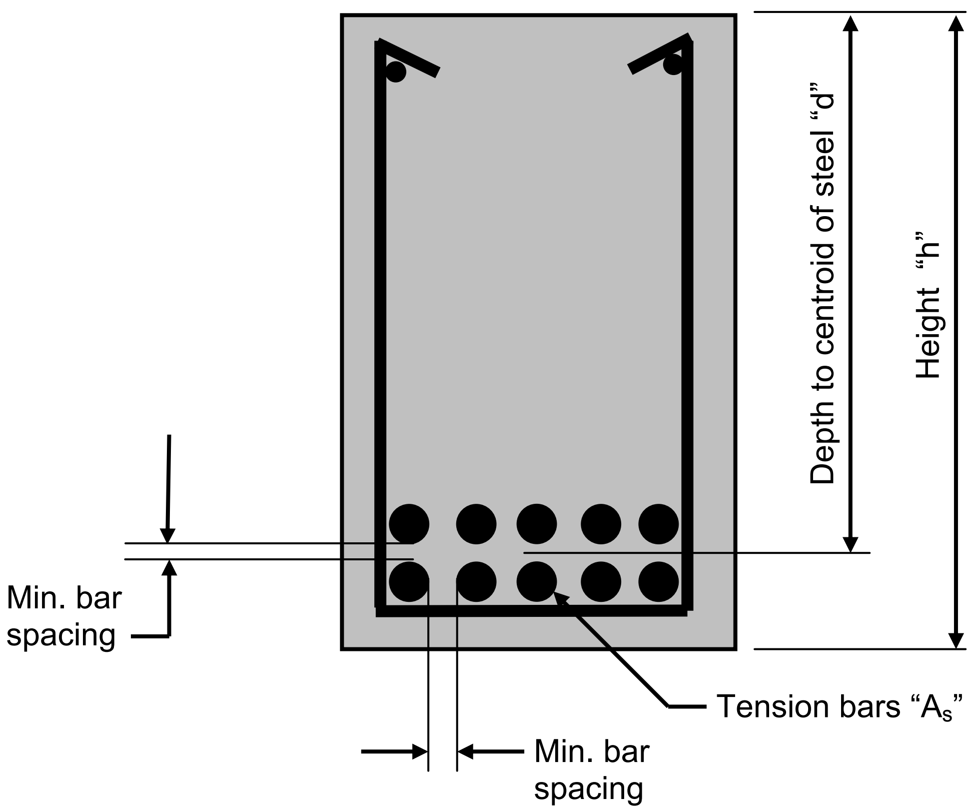




Section A-A

Sometimes, 2 (or more) rows of main tension bars are necessary. It is important to provide minimum adequate cover around all reinforcing bars so that these bars can properly bond with the concrete. ACI 318 dictates that the **minimum spacing between bars is 1.5 times the maximum concrete aggregate size**. Typical concrete batches use a maximum aggregate size of ¾" diameter, so then the minimum bar spacing = 1.5(¾") = 1⅛".

Below is a sketch of a typical concrete beam with **2 rows** of tension bars:



A_s = Total cross-sectional area of all tension bars, in^2

d = depth to center of tension bars, inches
= h – (concrete cover) – (stirrup bar dia.) – ½(tension bar dia.)

f_y = yield stress of reinforcing bars
= 60 KSI for ASTM A615 Grade 60 bars
= 40 KSI for ASTM A615 Grade 40 bars

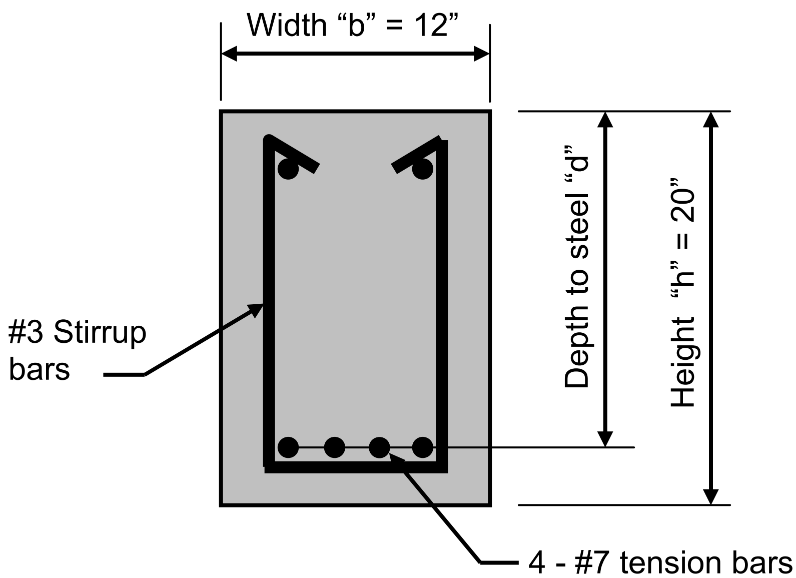
ρ_{actual} = Rho actual
= actual ratio of tension steel to effective concrete area
$= \frac{A_s}{bd}$

ρ_{min} = Rho minimum
= minimum allowable ratio of tension steel per ACI 318
$= \frac{200}{f_y}$ where f_y = PSI

Example 1

GIVEN: A rectangular concrete beam is similar to the one shown above. Use the following:

- Height h = 20”
- Width b = 12”
- Concrete f'_c = 4000 PSI
- Concrete cover = ¾”
- All bars are A615 – Grade 60 (f_y = 60 KSI)
- Stirrup bar = #3
- 4 - #7 Tension bars



REQUIRED:

1) Determine total area of tension bars, A_s.
2) Determine depth to center of tension bars, d.
3) Determine $\rho_{actual} = \frac{A_s}{bd}$ where $\rho_{min} = \frac{200}{f_y}$ and state if it is acceptable.

Step 1 – Determine area of tension bars, A_s:

A_s = 4 bars(0.60 in^2 per #7 bar) ← See Lect. 1 notes

$A_s = 2.40\ in^2$

Step 2 – Determine depth to tension bars, "d":

d = depth to center of tension bars, inches

= h – (concrete cover) – (stirrup bar dia.) – ½(tension bar dia.)

= 20" – ¾" – ⅜" – ½(⅞")

d = 18.44"

Step 3 – Determine ρ_{actual} and ρ_{min}:

$$\rho_{actual} = \frac{A_s}{bd}$$

$$= \frac{2.40 in^2}{(12")(18.44")}$$

ρ_{actual} = 0.0108

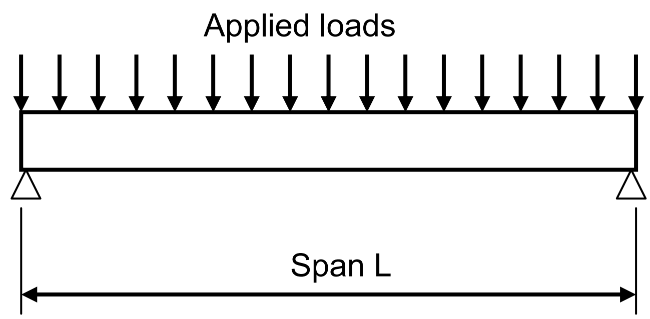
$$\rho_{min} = \frac{200}{f_y}$$

$$= \frac{200}{60000 PSI}$$

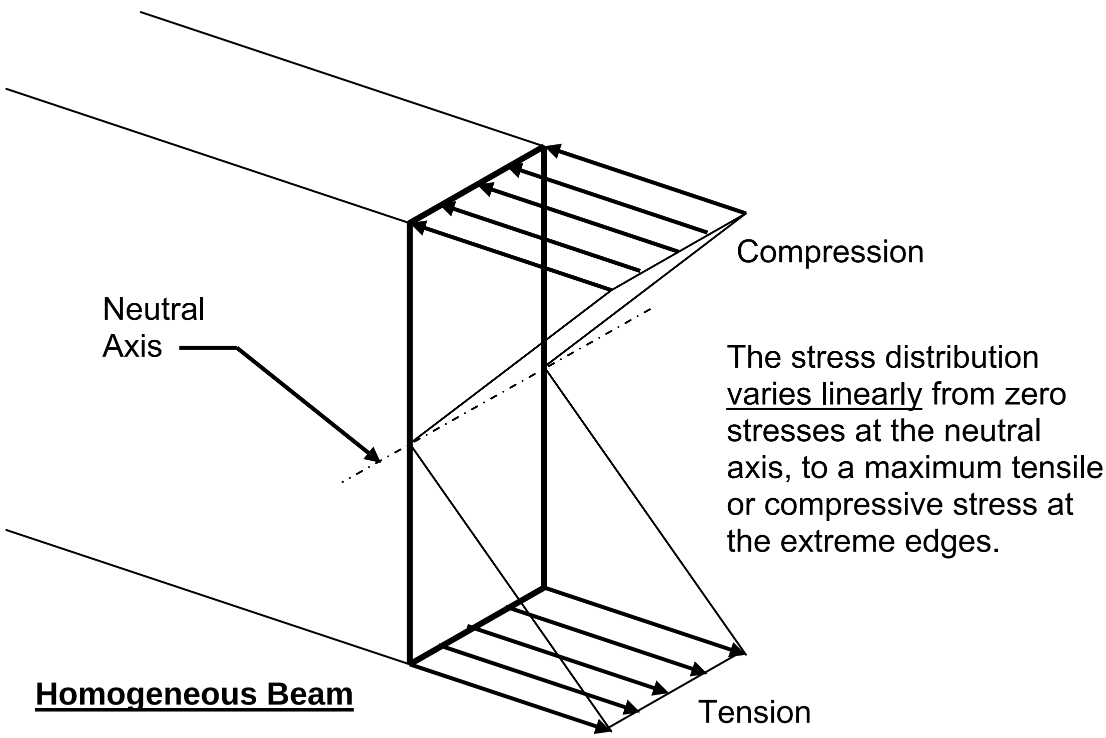
ρ_{min} = 0.0033

Since $\rho_{actual} > \rho_{min}$ → beam is acceptable

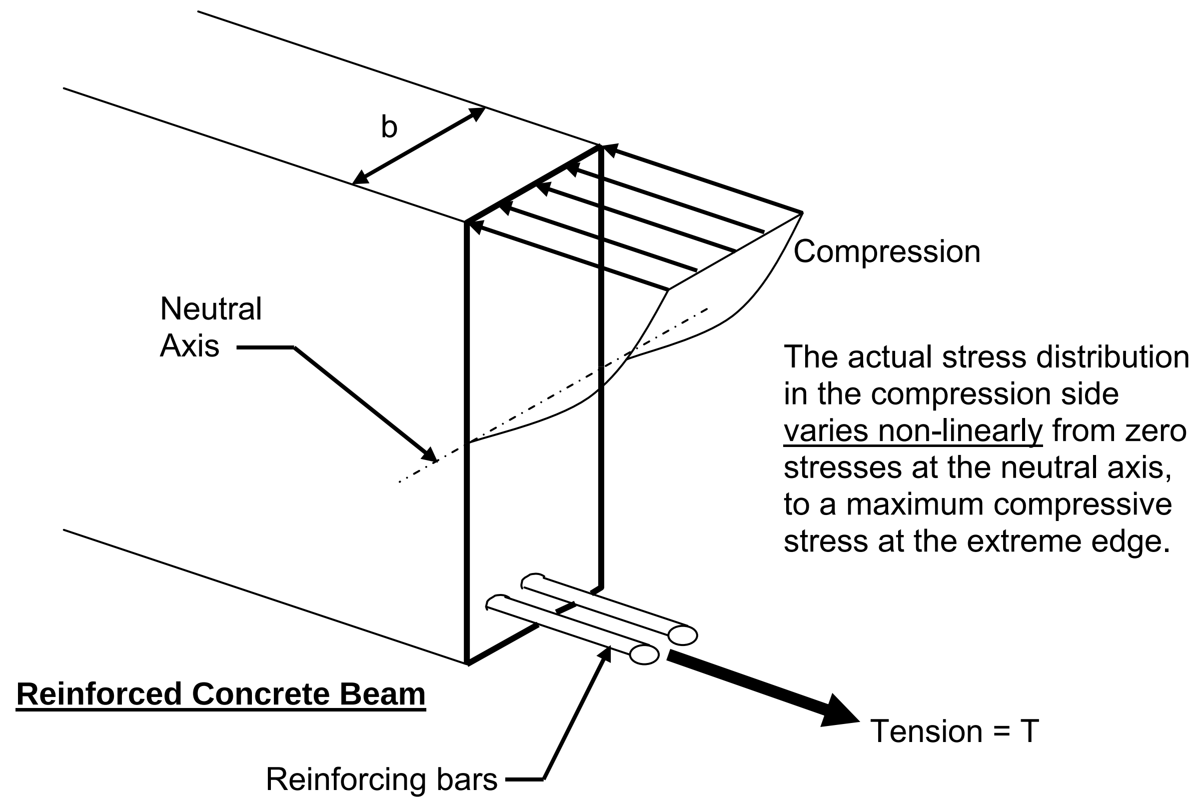
A basic understanding of beam mechanics is necessary to study concrete beam behavior. Consider a simply-supported homogeneous rectangular beam loaded by a uniformly-distributed load as shown below:



Taking a section through the beam at any place along the length reveals the following stress distribution about the cross-section of the beam:

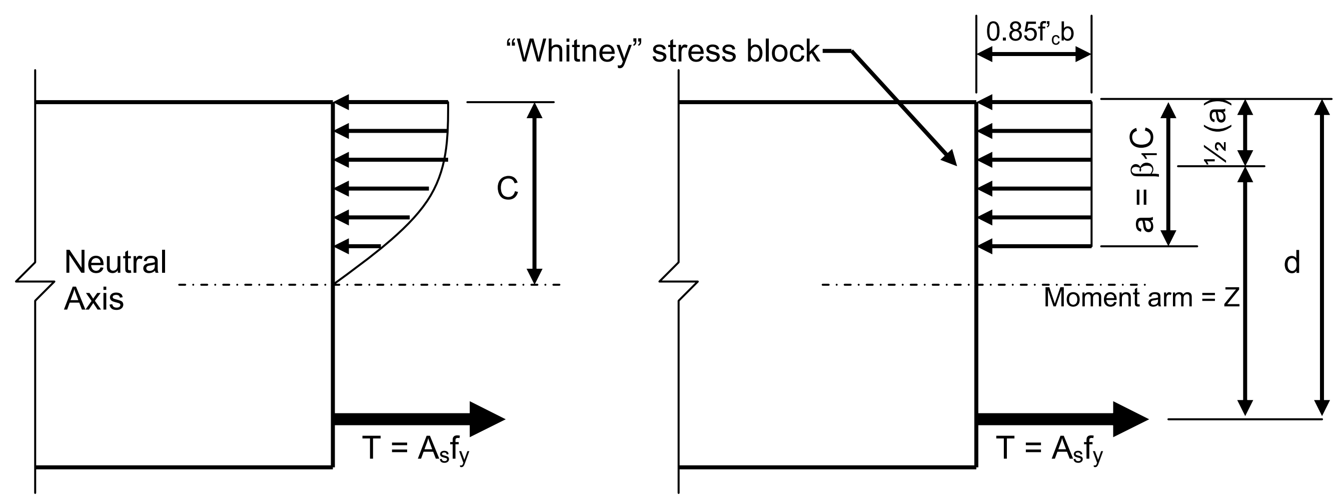


In a reinforced concrete beam, the stress distribution is different. Above the neutral axis, the concrete carries all the compression, similar to the homogeneous beam. Below the neutral axis however, the concrete is incapable of resisting tension and must rely on the reinforcing bars to carry all the tension loads.



Reinforced Concrete Beam

Looking at a side view of the stress distribution of the reinforced concrete beam:



Actual Stress Distribution **Idealized Stress Distribution**

Assuming an idealized beam, tension equals compression:

Tension = Compression
A_sf_y = Area of Whitney stress block
$A_sf_y = 0.85f'_cab$

Solve for a:

Beta

$$a = \frac{A_s f_y}{0.85 f'_c b} = \beta_1 C$$

β_1 = 0.85 for $f'_c \leq$ 4000 PSI
= 0.80 for f'_c = 5000 PSI
= 0.75 for $f'_c \geq$ 6000 PSI

C = depth to neutral axis from extreme compression edge

M_n = Nominal moment capacity of concrete beam
= A_sf_y(Moment arm)
= A_sf_yZ
= $A_sf_y(d - \frac{a}{2})$

M_u = Usable moment capacity of concrete beam
= ϕM_n
= $0.9M_n$

$M_u = 0.9(A_sf_y(d - \frac{a}{2}))$

$$\mathbf{M_u = 0.9A_sf_yd\left(1 - \left[0.59\left(\frac{\rho_{act} f_y}{f'_c}\right)\right]\right)}$$

ρ_{bal} = balanced ratio of tension steel reinforcement

$$= \left(\frac{0.85\beta_1 f'_c}{f_y}\right)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$ where f_y = PSI

ρ_{max} = maximum allowable ratio of tension steel reinforcement per ACI 318
= $0.75\rho_{bal}$

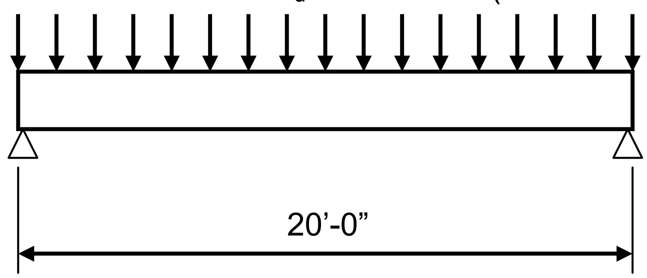
Example 2

GIVEN: The concrete beam from Example 1 is used to support the loading as shown below.

REQUIRED:

1. Determine the maximum factored applied moment, M_{max}.
2. Determine the usable moment capacity of the beam, M_u, and determine if it is acceptable based on M_{max}.
3. Determine if the beam is acceptable based on ρ_{max}.

Factored uniform load w_u = 3000 PLF (incl. beam wt.)



Step 1 – Determine maximum factored applied moment, M_{max}:

$$M_{max} = \frac{w_u L^2}{8}$$

$$= \frac{(3KLF)(20'\text{–}0")^2}{8}$$

M_{max} = 150 KIP-FT

Step 2 - Determine the usable moment capacity of the beam, M_u:

$$M_u = 0.9 A_s f_y d\left(1 - \left[0.59\left(\frac{\rho_{act} f_y}{f'_c}\right)\right]\right)$$ where ρ_{act} = 0.0108 (see Ex. 1)

$$= 0.9(2.40\ in^2)(60\ KSI)(18.44")\left(1 - \left[0.59\left(\frac{(0.0108)(60KSI)}{4KSI}\right)\right]\right)$$

= 2161.4 KIP-IN

M_u = 180.1 KIP-FT

Since M_u = 180.1 KIP-FT > M_{max} = 150 KIP-FT → beam is acceptable

Step 3 – Determine if the beam is acceptable based on ρ_{max}:

ρ_{max} = maximum allowable ratio of tension steel reinforcement per ACI 318
= $0.75\rho_{bal}$

ρ_{bal} = balanced ratio of tension steel reinforcement

$$= \left(\frac{0.85\beta_1 f'_c}{f_y}\right)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$ where f_y = PSI

where $\beta_1 = 0.85$ since $f'_c = 4000$ PSI

$$= \left(\frac{0.85(0.85)(4KSI)}{60KSI}\right)\left(\frac{87{,}000}{87{,}000 + 60000PSI}\right)$$

= 0.0285

$\rho_{max} = 0.75(0.0285)$

$\rho_{max} = 0.0214 > \rho_{act} = 0.0108 \rightarrow$ beam is acceptable