Lecture 1 – Review of Units

Basic Fundamental Units						
	US Units:			**Metric Units:**		
Length (L)	inch	foot	yard	mm	cm	m
Force (P)	pounds	1 Kip = 1000 lbs.	ton	Newtons (N)	kN	

Note: 1 inch = 25.4 mm
1 pound = 4.448 newtons (N)
1 kg = 9.81 N

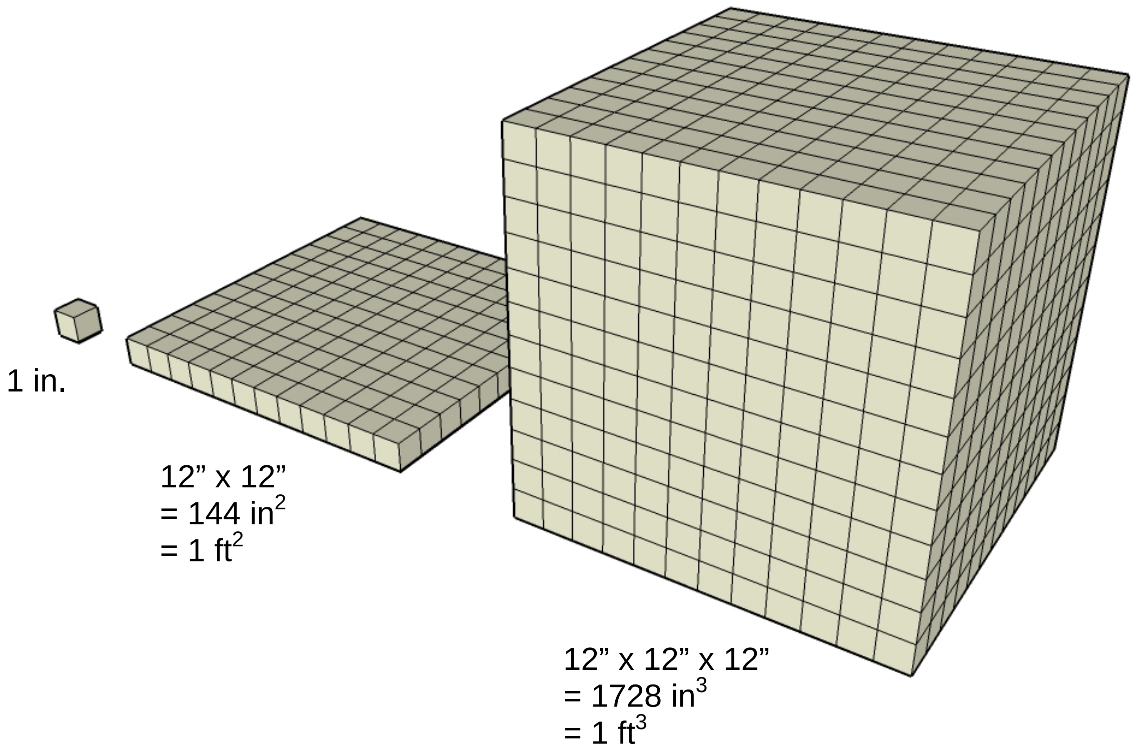
1 lb. = 0.454 kg

Combinations of Units						
	Basic Formula:	**US Units:**			**Metric Units:**	
Area (A)	Length x Length	in^2	ft^2	yd^2	mm^2	m^2
Volume (V)	Length x Length x Length	in^3	ft^3	yd^3	mm^3	m^3
Unit Weight (γ)	$\frac{Force}{Volume}$	$\frac{lb}{in^3}$	$\frac{lb}{ft^3}$	$\frac{kip}{ft^3}$	$\frac{N}{mm^3}$	$\frac{kN}{m^3}$
Uniform Load (w) **Linear Load**	$\frac{Force}{Length}$	$\frac{lb}{ft}$	$\frac{lb}{in}$	$\frac{kip}{ft}$	$\frac{N}{mm}$	$\frac{kN}{m}$
Stress (f) **Pressure (σ)**	$\frac{Force}{Area}$	$\frac{lb}{in^2}$	$\frac{lb}{ft^2}$	$\frac{kip}{in^2}$	$\frac{N}{m^2} = Pa$	$\frac{kN}{mm^2}$
Strain (ε)	$\frac{Length}{Length}$	$\frac{in}{in}$			$\frac{mm}{mm}$	
Modulus of Elasticity (E)	$\frac{Force}{Area}$	$\frac{lb}{in^2}$	$\frac{lb}{ft^2}$	$\frac{kip}{in^2}$	$\frac{N}{m^2} = Pa$	$\frac{kN}{mm^2}$
Moment (M) **Torque**	$(Force)(Length)$	$ft-lb$	$kip-ft$	$kip-in$	$N-mm$	$kN-m$
Moment of Inertia (I)	$(Length)^4$	in^4			mm^4	
Section Modulus (S)	$(Length)^3$	in^3			mm^3	
Radius of Gyration (r)	$\sqrt{\frac{I}{A}}$	in			mm	
Deflection (Δ)	Long & nasty formulas	in			mm	
Deformation (δ)	$\frac{PL}{AE}$	in			mm	

Note: 1 psf = 47.88 Pa
1 ksi = 6.895 $\frac{N}{mm^2} = 6.895 MPa$

1 m^3 = 35.3 ft^3

It is good to think of the relationship between linear, squared and cubic units 3-dimensionally as shown below:



Greek Letters

Upper Case:	Lower Case:	Name:
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	φ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

Dimensional Analysis (Unit Conversions)

Perhaps the *most important mathematical strength* is the ability to perform "Dimensional Analysis".

> Dimensional analysis is converting one set of units into another.

The most basic dimensional analysis would involve relating one single unit to another single unit. A simple conversion of length such as inches to feet would be an example. Or time conversion such as days to hours.

Dimensional analysis is achieved by multiplying fractions in order to obtain the desired end units. The following steps can be used:

a) Write the initial unit in fraction form on the left.
b) Write the final desired unit in fraction form on the right.
c) Working from left to right, write down conversions of initial units to final units.
d) Cancel numerators with denominators to reduce all fractions into final unit fraction.

If initial unit fraction = $\frac{a}{b}$

If final desired unit fraction = $\frac{y}{z}$

$$\frac{a}{b} = \left(\frac{a}{b}\right)\left(\frac{b}{c}\right)\left(\frac{c}{a}\right)\left(\frac{d}{e}\right)\cdots\left(\frac{y}{z}\right)$$

↓ Cancel numerators with denominators

$$\frac{a}{b} = \left(\frac{\cancel{a}}{\cancel{b}}\right)\left(\frac{\cancel{b}}{\cancel{c}}\right)\left(\frac{\cancel{c}}{\cancel{a}}\right)\left(\frac{d}{e}\right)\cdots\left(\frac{y}{z}\right)$$

↓ Obtain final unit

$$\frac{a}{b} = \frac{y}{z}$$

Example 1
GIVEN: A wood board is five feet long.
REQUIRED: Determine the length of the board in units of inches. Use the conversion tables above.

Initial unit fraction = $\frac{a}{b}$ = 5 feet = $\frac{5\,feet}{1}$

Final desired unit fraction = $\frac{y}{z}$ = inches = $\frac{inches}{1}$

5 feet = $\frac{5\,feet}{1}$ x $\frac{12 inches}{1\,foot}$

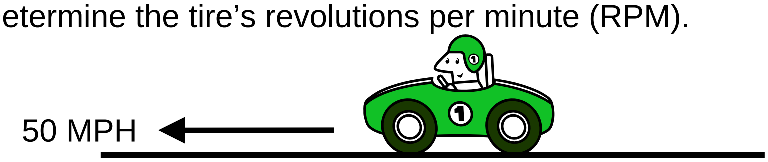
= $\frac{5\,\cancel{feet}}{1}$ x $\frac{12 inches}{1\,\cancel{foot}}$

= $\frac{5x12xinches}{1x1}$

= $\frac{60 inches}{1}$

= **60 inches**

Example 2
GIVEN: A car drives 50 miles per hour (MPH), and has tires with an outside diameter of 24”.
REQUIRED:
1) Determine the car’s velocity in units of feet per second.
2) Determine the tire’s revolutions per minute (RPM).



$$\frac{50 Miles}{1\_Hour} x \frac{1\_Hour}{60\_Min} x \frac{1\_Min}{60\_Sec} x \frac{5280 Feet}{1\_Mile} = \boxed{73.3 \frac{Feet}{Sec}}$$

1 revolution = circumference
= πD
= π(2 feet)
= 6.28 feet

$$\frac{1\_\mathrm{Re}v}{6.28 Feet} x \frac{73.3 Feet}{1\_Sec} x \frac{60 Sec}{1\_Min} = \boxed{700 \frac{\mathrm{Re}v}{Min}}$$

Example 3

GIVEN: A structural steel beam using ASTM A992 material has a yield stress of 50 KSI.

REQUIRED:

Determine the yield stress in units of MPa.

1 MPa = 1,000,000 Pa

$= 1{,}000{,}000\ \frac{N}{m^2}$

Set up relationships:

$$\frac{50Kips}{in^2} x \frac{(1in)^2}{(25.4mm)^2} x \frac{4.448N}{1\_lb} x \frac{1000lb}{1\_Kip} = 344.7\frac{N}{mm^2}$$

Recall:

1000 mm = 1 m

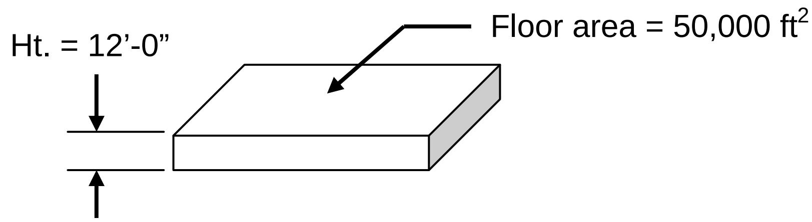
$1{,}000{,}000\ mm^2 = 1\ m^2 \rightarrow 1\ MPa = \frac{1{,}000{,}000N}{m^2} = \frac{1N}{mm^2}$

$$\boxed{344.7\frac{N}{mm^2} = 344.7MPa}$$

Example 4

GIVEN: A high-tech industrial building having a floor area = 50,000 ft^2 and wall height = 12'-0" must have continuous air changes at a minimum rate of 3 complete air changes per minute (i.e., 3 entire building volumes of air per minute).

REQUIRED: How many HVAC air-handlers must be used if each air-handling unit is capable of moving 100 m^3 per second?



Volume = Area x Height
= (50,000 ft^2)(12 ft)
= 600,000 ft^3

Initial unit fraction = 3 x $\frac{600{,}000\, ft^3}{1 \min}$

$$= \frac{900{,}000\, ft^3}{1 \min}$$

Final desired unit fraction = $\frac{m^3}{1 \sec}$

$$\frac{900{,}000\, ft^3}{1 \min} = \frac{900{,}000\, ft^3}{1 \min} \times \frac{1 m^3}{35.3\, ft^3} \times \frac{1 \min}{60 \sec}$$

$$= \frac{900{,}000\, \cancel{ft^3}}{1 \cancel{\min}} \times \frac{1 m^3}{35.3\, \cancel{ft^3}} \times \frac{1 \cancel{\min}}{60 \sec}$$

$$= \frac{(900{,}000)(1 m^3)(1)}{(1)(35.3)(60 \sec)}$$

$$= \frac{425 m^3}{\sec}$$

Number of air handlers required = $\frac{425 m^3}{\sec} \times \frac{1\_airhandler}{\frac{100 m^3}{\sec}}$ = 4.25 air handlers

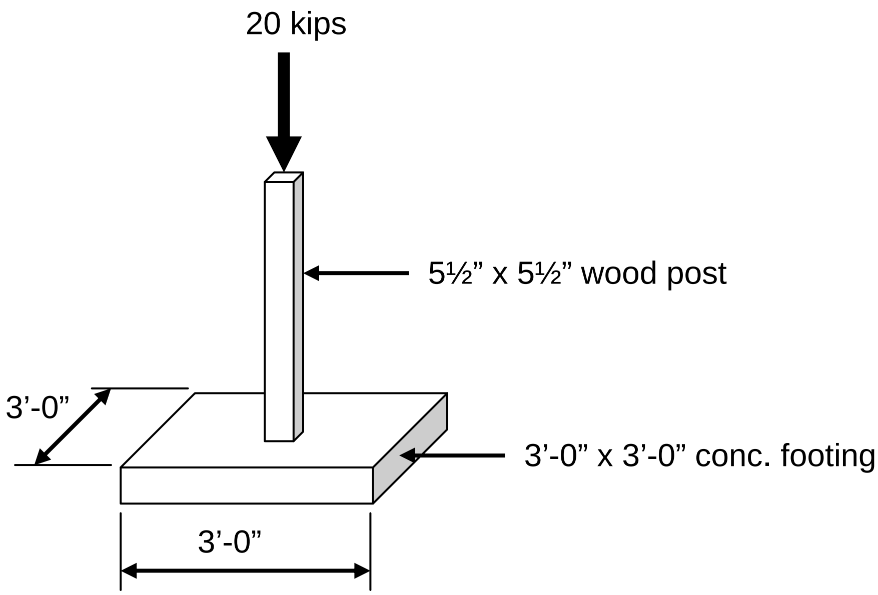
USE 5 air handlers

Example 5

GIVEN: A nominal 6X6 wood post (actual size = 5½" x 5½") supports a load of 20 kips. The post is supported by a 3'-0" x 3'-0" square concrete footing.

REQUIRED:

1) If the maximum permissible load on the post = 450 PSI, determine if the post is acceptable.
2) If the maximum permissible soil bearing pressure = 1.5 tons per square foot, determine if the concrete footing size is adequate. Assume the footing itself weighs 1350 lbs.



Part 1:

Initial unit fraction $= \dfrac{20kips}{(5.5")(5.5")}$

Final desired unit fraction $= \dfrac{lbs}{in^2}$

$$\frac{20kips}{(5.5")(5.5")} = \frac{20\cancel{kips}}{(5.5")(5.5")} \times \frac{1000lbs.}{1\cancel{kip}}$$

$$= \frac{20000lbs.}{30.25in^2}$$

$$= \frac{661lbs.}{in^2}$$

→ **Post is unacceptable since 661 PSI is greater than 450 PSI.**

<u>Part 2</u>:

Total weight to be supported by soil = column load + footing weight
= 20 kips + 1350 lbs.
= 20,000 lbs. + 1350 lbs.
= 21,350 lbs.

Initial unit fraction = $\dfrac{21{,}350 lbs.}{(3'-0'')(3'-0'')}$

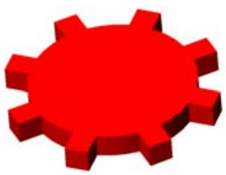
Final desired unit fraction = $\dfrac{tons}{ft^2}$

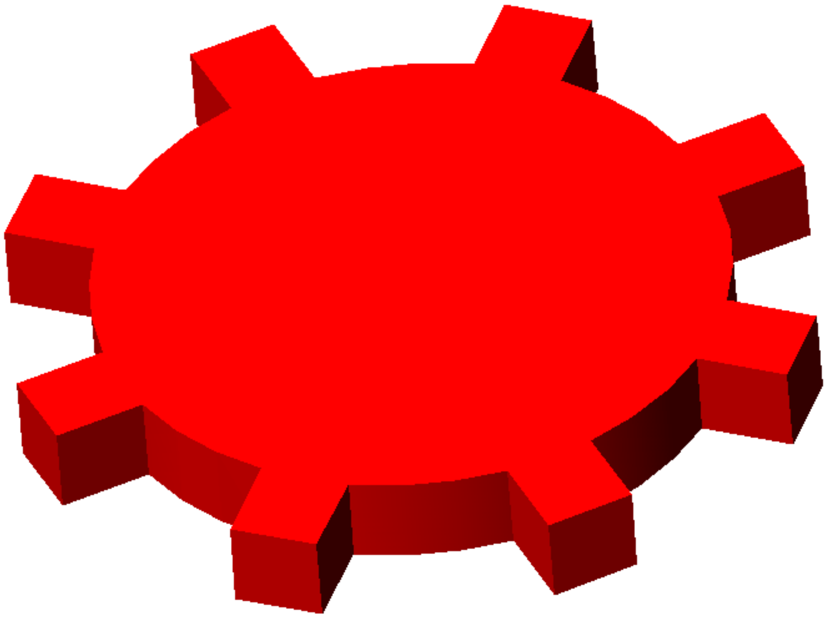
$$\frac{21{,}350 lbs.}{(3'-0'')(3'-0'')} = \frac{21{,}350 \cancel{lbs.}}{(3'-0'')(3'-0'')} \times \frac{1 ton}{2000 \cancel{lbs.}}$$

$$= \frac{1.18 ton}{ft^2}$$

→ **Footing is acceptable since 1.18 tons per ft^2 is less than 1.5 tons per ft^2.**

Example 6
GIVEN: A ¼ scale model steel gear weighs 7 lbs.
REQUIRED: How much does a full-scale part, assuming using the exact same material and proportion?





¼ scale = 7lbs. Full-scale → wt. = ???

Since the object is 3-dimensional, the weight is proportional to the CUBE of the dimensions.

Set up a proportion:

$$\frac{Model\_weight}{LxWxH} = \frac{Full\_Scale\_weight}{LxWxH}$$

Assuming each dimension of the model is ¼ of the real, then:

$$\frac{7\_Lbs.}{\frac{1}{4}x\frac{1}{4}x\frac{1}{4}} = \frac{Full\_Scale\_weight}{1x1x1}$$

By cross-multiplication,

$$Full\_Scale\_weight = \frac{7\_Lbs.}{\frac{1}{4}x\frac{1}{4}x\frac{1}{4}}$$

Full-Scale weight = 448 Lbs.