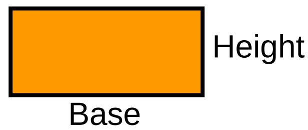
Lecture 4 – Section Properties (symmetric shapes)

1. Area

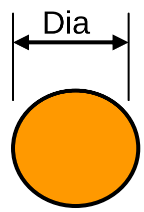
Area has units of length squared – typically in^2, ft^2, mm^2, etc.

a) Rectangle:

Area = base x height



b) Circle:



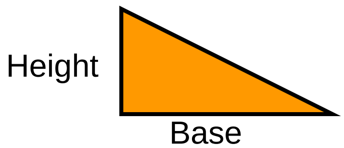
Area = $\frac{\pi}{4} Dia^2$

Circumference = πD

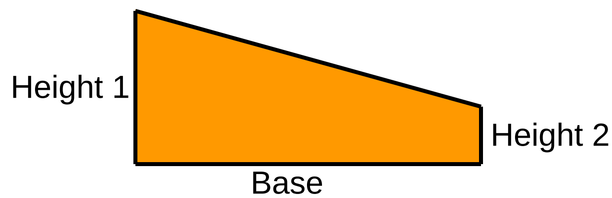
Hollow circle area = A_{outer} - A_{inner}

c) Triangle:

Area = ½(Base)(Height)



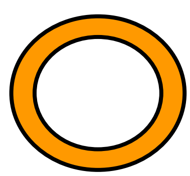
d) Trapezoid:



Area = ½(Height 1 + Height 2)(Base)

e) Hollow shapes:

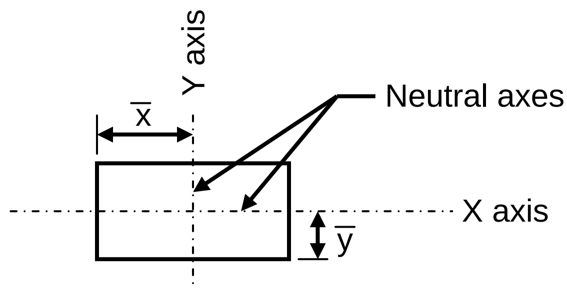
Area = A_{outer} - A_{inner}



2. Location of Centroid

The centroid of a shape is the **location of the center of its weight**. It is the point on a shape where you could balance it in equilibrium. It is the location of the intersection of the neutral axes in the primary X and Y axes. The location of the centroid is essential for determining other cross-sectional properties such as moment of inertia.

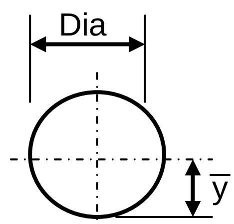
a) **Rectangle**:



$\bar{y}$ = ½(Height)

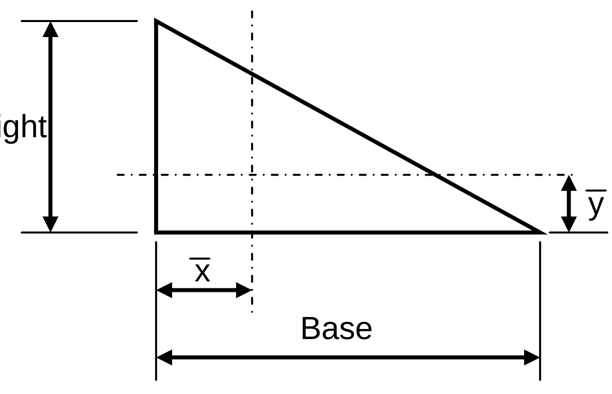
$\bar{x}$ = ½(Base)

b) **Circle**:



$\bar{y}$ = ½(Dia)

c) **Triangle**:



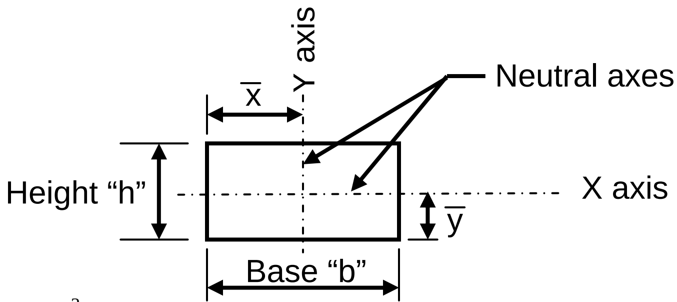
$\bar{y} = \frac{1}{3}$(Height)

$\bar{x} = \frac{1}{3}$ (Base)

3. Moment of Inertia

The moment of inertia, or "I" is a **measure of the stiffness** of the cross-sectional shape. It is the property which explains why, for example, a 2x10 wood joist is oriented vertically rather than flat to carry more load. It is sometimes referred to the second moment of area. The moment of inertia has units of length raised to the 4^{th} power – typically in^4 of mm^4. The moment of inertia is typically measured about the strong axis (usually the X axis) and the weak axis (usually the Y axis).

a) Rectangle:



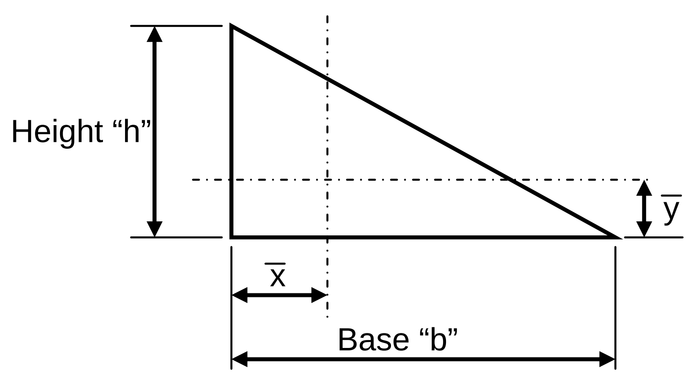
$$I_x = \frac{bh^3}{12}$$ where b = base and h = height

$$I_y = \frac{hb^3}{12}$$

b) Circle:

$$I_x = I_y = \frac{\pi D^4}{64}$$ where D = Diameter

c) Triangle:



$$I_x = \frac{bh^3}{36}$$

d) Symmetric Hollow Shapes:

$I = I_{outer} - I_{inner}$

4. Section Modulus

The section modulus or "S" is **also a measure of the stiffness** of a cross-sectional shape. It is sometimes more convenient to use this rather than moment of inertia for calculations such as bending stresses. It has units of length raised to the 3rd power – typically in^3 or mm^3. Do NOT get it confused with volume!! Similar to moment of inertia, the section modulus is typically measured about the strong axis (usually the X axis) and the weak axis (usually the Y axis).

General formula for determining section modulus:

$$S_x = \frac{I_x}{y}$$ where I_x = moment of inertia about x axis

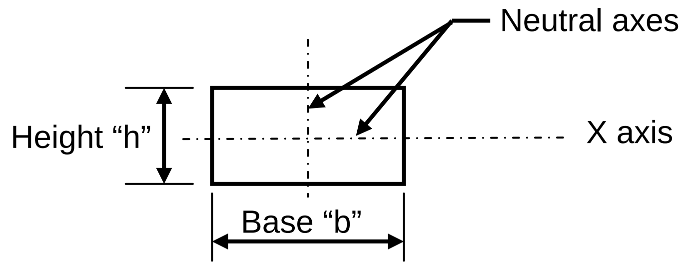
y = distance to neutral axis

$$S_y = \frac{I_y}{x}$$ where I_y = moment of inertia about y axis

x = distance to neutral axis

a) Rectangle:

$$S_x = \frac{\left(\frac{bh^3}{12}\right)}{\left(\frac{h}{2}\right)} = \frac{bh^2}{6}$$



$$S_y = similar$$

b) Other shapes:

Section modulus shall be determined as dividing moment of inertia by distance to neutral axis as mentioned above.

5. Radius of Gyration

The radius of gyration is used most often in column design as a measure of a cross-sectional shape's **ability to resist buckling**. It has units of length – typically inches or mm.

General formula for determining radius of gyration:

$$r_x = \sqrt{\frac{I_x}{A}}$$ where I_x = moment of inertia about x axis

A = cross-sectional area

$$r_y = \sqrt{\frac{I_y}{A}}$$ where I_y = moment of inertia about y axis

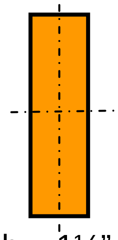
A = cross-sectional area

Example 1:

GIVEN: A wood 2x8 (actual 1½" x 7¼")

REQUIRED: Determine

a) Area
b) Location of centroid
c) Moment of inertia about strong and weak axes
d) Section modulus about strong and weak axes
e) Radius of gyration about strong **and** weak axes.



a) Area:

A = bh
= (1.5")(7.25")
= **10.88 in²**

b) Location of centroid:

$\bar{y}$ = ½(Height)
= ½(7.25")
= **3.625"**

$\bar{x}$ = ½(Base)
= ½(1.5")
= **0.75"**

c) Moment of inertia:

$$I_x = \frac{bh^3}{12}$$ where b = base and h = height

$$= \frac{(1.5")(7.25")^3}{12}$$

= **47.63 in⁴ Strong axis**

$$I_y = \frac{hb^3}{12}$$

$$= \frac{(7.25")(1.5")^3}{12}$$

= **2.04 in⁴ Weak axis**

d) **<u>Section modulus</u>:**

$$S_x = \frac{bh^3/12}{h/2} = \frac{bh^2}{6}$$

$$S_x = \frac{(1.5")(7.25")^3/12}{7.25"/2} = \frac{(1.5")(7.25")^2}{6}$$

= **$13.14\ in^3$ Strong axis**

$$S_y = \frac{hb^3/12}{b/2} = \frac{hb^2}{6}$$

$$S_y = \frac{(7.25")(1.5")^3/12}{1.5"/2} = \frac{(7.25")(1.5")^2}{6}$$

= **$2.72\ in^3$ Weak axis**

e) **<u>Radius of gyration</u>:**

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$r_x = \sqrt{\frac{47.63 in^4}{10.88 in^2}}$$

= **2.09 in. Strong axis**

$$r_y = \sqrt{\frac{I_y}{A}}$$

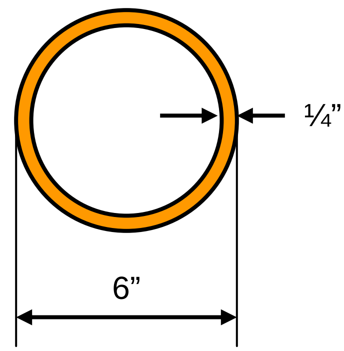
$$r_y = \sqrt{\frac{2.04 in^4}{10.88 in^2}}$$

= **0.43 in. Weak axis**

Example 2:

GIVEN: A 6" O.D. hollow steel pipe with ¼" wall thickness
REQUIRED: Determine

a) Area
b) Location of centroid
c) Moment of inertia about strong and weak axes
d) Section modulus about strong and weak axes
e) Radius of gyration about strong and weak axes.



a) Area:

$A_{tot} = A_{out} - A_{in}$

$$= \left(\frac{\pi}{4} Dia^2\right)_{outer} - \left(\frac{\pi}{4} Dia^2\right)_{inner}$$

$$= \left(\frac{\pi}{4}(6")^2\right)_{outer} - \left(\frac{\pi}{4}(5\frac{1}{2}")^2\right)_{inner}$$

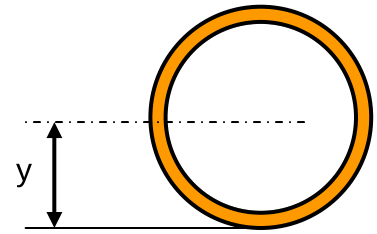
$= 28.27 \text{ in}^2 - 23.76 \text{ in}^2$

$A_{tot} = 4.51 \text{ in}^2$

b) Location of Centroid:

Centroid location "y" = $\frac{1}{2} O.D.$

$= \frac{1}{2}(6")$



Centroid location "y" = 3" from bottom of pipe

c) **Moment of Inertia about Strong & Weak Axis:**

$$I_{tot} = I_{out} - I_{in}$$

$$= \frac{\pi D_{out}^{\ 4}}{64} - \frac{\pi D_{in}^{\ 4}}{64}$$ where D = Diameter

$$= \frac{\pi(6")^4}{64} - \frac{\pi(5\frac{1}{2}")^4}{64}$$

= 63.62 in^4 – 44.92 in^4

I_{tot} = 18.70 in^4

d) **Section Modulus about Strong & Weak Axis:**

$$S = \frac{I_{tot}}{y}$$

$$= \frac{18.70 in^4}{3"}$$

S = 6.23 in^3

e) **Radius of Gyration about Strong & Weak Axis:**

$$r_x = \sqrt{\frac{I_{tot}}{A_{tot}}}$$

$$= \sqrt{\frac{18.70 in^4}{4.51 in^2}}$$

r = 2.04 in